

NAME:

Math 230H Test 4: Thursday November 15th

The test consists of four questions, each of which will be graded out of 10 points. Show all your working. Calculators are not permitted.

1. Let S denote the square $\{(u, v) : 1 \leq u, v \leq 2\}$ in the uv -plane. The region R of the xy -plane is obtained from S by the coordinate transformation

$$x = uv, \quad y = \ln(v/u).$$

Find the area of R .

2. A wire of density $\delta(x, y, z) = 15\sqrt{y+2}$ lies along the curve $\mathbf{r}(t) = (t^2 - 1)\mathbf{j} + 2t\mathbf{k}$, $-1 \leq t \leq 1$. Find its center of mass.

3. Evaluate $\int_C (x-y)dx + (x+y)dy$ counterclockwise around the triangle with vertices $(0, 0)$, $(1, 0)$, and $(0, 1)$. [You may use Green's theorem if you wish.]

4. Show that the vector field $\mathbf{F} = e^{y+2z}(\mathbf{i} + x\mathbf{j} + 2x\mathbf{k})$ is conservative.
Evaluate

$$\int_C \mathbf{F} \cdot d\mathbf{r},$$

where C is the semicircular arc, center the origin and radius 5, from $(0, -5, 0)$ through $(3, 0, 4)$ to $(0, 5, 0)$.