PING XU

Research Statement

The main thrust of my research in the past has been focused on the study of problems in mathematical physics, which are related to Poisson geometry and noncommutative geometry.

Poisson geometry. Poisson geometry originated in the last century in the Hamilton-Jacobi formulation of classical mechanics using what is now called Poisson brackets. It became formalized in the language of modern differential geometry about 40 years ago. This gave rise to Poisson manifolds and Poisson algebras. The quantization problem -passage from classical mechanics to quantum mechanics- is nowadays often formulated in terms of Poisson manifolds. An important class of Poisson manifolds is given by the phase spaces of classical mechanics. These phase spaces are symplectic manifolds, and the study of these manifolds, called symplectic geometry, has become a huge field in modern mathematics, both pure and applied. However there are many important Poisson manifolds which are not symplectic. For instance the target space of a momentum map is a Poisson manifold, and the quotient of a symplectic manifold by a Hamiltonian group action is a Poisson manifold. Other important examples are Poisson-Lie groups, which are important in the theory of integrable systems, and whose quantization gives quantum groups. My main interest is in the geometric structure and quantization of Poisson manifolds with applications in integrable systems and noncommutative geometry. Among the main tools are Lie groupoids and Lie algebroids.

My work with Liu and Weinstein on Poisson geometry coined the notion of "Courant algebroids" [48], which has become a fundamental concept in the recent development of so called "generalized geometry" of Hitchin and his school. Recently "generalized geometry" has become a very active area of research in mathematical physics due to its close connection with string theory. The theory of Courant algebroids was developed as a Lie algebroid analogue of Manin triples. One of the motivation was to unify the theory of Dirac structures of Courant (these include closed 2-forms, Poisson structures, and foliations) with Drinfeld's theory of Poisson homogeneous spaces. Liu, Weinstein and I in particular classified Poisson homogeneous spaces for Poisson groupoids in terms of Dirac structures for the corresponding Lie bialgebroids [46]. As an application, we obtained a new proof of Drinfeld's theorem concerning Poisson homogeneous spaces, and gave a more geometric explanation of his result.

Courant algebroids are, in a certain sense, infinitesimal objects associated with gerbes, which are also related to L_∞-algebras. There are some new developments in Courant algebroids recently. For example, recent work of Severa and Weinstein shows that Courant algebroids are related to string theory and D-branes. In a recent joint work, Alekseev and I proved that any Courant algebroid is generated by a Dirac-type operator with a cubic term. In some special cases, such a Dirac generating operator is related to equivariant cohomology.

In the C. R. Acad. Note [16], Stienon and I introduced the naive cohomology and the modular class of a Courant algebroid, as invariants of Courant algebroids. These invariants are much easier to handle than the standard cohomology of a Courant algebroid. We conjectured that the naive and standard cohomologies are isomorphic when the Courant algebroid is transitive. This conjecture was recently verified by Ginot and Grutzmann in a paper published in J. of Symplectic Geometry.

In order to understand the intrinsic connection between the Poisson group theory and the theory of symplectic groupoids, Mackenzie and I developed the theory of Lie bialgebroids [54]. As a continuation, in our second paper [37], we solved the integration problem for general Lie bialgebroids, which extends the well-known result of Drinfel'd that a Lie bialgebra is the Lie bialgebra of a Poisson group. As an application, we obtained a new proof of the existence of local symplectic groupoids for any Poisson manifolds, a remarkable theorem of Karasev and Weinstein. Our results elucidate the origin of the groupoid structure and symplectic structure on a symplectic groupoid. Given a Poisson manifold P, its cotangent bundle T^*P carries a Lie algebroid structure. The canonical Lie algebroid structure on its dual, that is, the tangent bundle TP, induces a Poisson structure on its groupoid which happens to be symplectic in this case. The compatibility condition between these two Lie algebroid structures assures the compatibility condition between the groupoid and symplectic structures which makes it into a symplectic groupoid. Some important properties of Poisson groupoids were studied in [51]. In particular, I proved the multiplicativity condition for the Poisson tensor on an arbitrary Poisson groupoid, which had been a long standing question even for symplectic groupoids. A general study on multiplicative multivector fields and forms on Lie groupoids was carried out in [45] with Mackenzie.

Lie bialgebroids are particularly useful in studying generalized complex geometry. In fact, a generalized complex structure is equivalent to a (complex) Lie bialgebroid where one Lie algebroid is a complex conjugate of the other. This viewpoint leads to many fruitable results in generalized complex geometry. For instance, Stienon and I obtained the reduction result for generalized complex manifolds in [17], which are also independently obtained by another two groups of mathematicians. Stienon and I also introduced the notion of Poisson quasi-Nijenhuis manifolds, generalizing the Poisson-Nijenhuis manifolds of Magri-Morosi. We also investigate the integration problem of Poisson quasi-Nijenhuis manifolds. As a result, we show that a generalized complex structure integrates to a symplectic quasi-Nijenhuis groupoid, recovering a theorem of Crainic.

In [23], I developed the theory of quasi-symplectic groupoids and their momentum map theory. This theory enables us to unify into a single framework various momentum map theories, including ordinary Hamiltonian G-spaces, the momentum map of Poisson group actions, and the group-valued momentum map of Alekseev–Malkin–Meinrenken. With Laurent-Gengoux, we applied this idea of momentum map to quantization of quasi-presymplectic groupoids and their Hamiltonian spaces. As an application, we studied the prequantization of the quasi-Hamiltonian G-spaces of Alekseev–Malkin–Meinrenken, and recovered Alekseev-Meinrenken's integrality condition of a quasi-Hamiltonian G-space. This unified momentum map was recently further studied by Zung in connection with the convexity.

With Iglesias and Laurent-Gengoux, I studied the "integration problem" in Poisson geometry from a general perspective and proved the so called "universal lifting theorem": on an s-simply connected and s-connected Lie groupoid Γ with Lie algebroid A, the graded Lie algebra of multi-differentials on A is isomorphic to that of multiplicative multi-vector fields on Γ. This theorem gives an intrinsic explanation for the origin of various integration theorems in the literature. In particular, as a consequence, we obtain an integration theorem for quasi-Lie bialgebroids. We also initiated a systematic study of basic properties of quasi-Poisson groupoids. In particular, we prove that, given a pair of group (D, G) associated to a Manin quasi-triple $(\mathfrak{d}, \mathfrak{g}, \mathfrak{h})$, the transformation groupoid $G \rtimes D/G \rightrightarrows D/G$ is endowed with a quasi-Poisson structure. Its momentum map corresponds exactly to the D/G-momentum map of Alekseev and Kosmann-Schwarzbach. Recently, Chen, Stienon and I proved a 2-group version of universal lifting theorem [1], and, as a consequence, proved that there is a bijection between Poisson 2-groups and Lie 2-bialgebras.

In [13], Laurent-Gengoux, Stienon and I solved the integration problem of holomorphic Lie algebroids. More precisely, we proved that a holomorphic Lie algebroid is integrable if, and only if, its underlying real Lie algebroid is integrable. Thus the integrability criteria of Crainic-Fernandes also apply in the holomorphic context without any modification. As a consequence we give another proof of the following theorem: a holomorphic Poisson manifold is integrable if, and only if, its real (or imaginary) part is integrable as a real Poisson manifold.

Lie bialgebroids are now known to be connected with various other geometric structures, such as Poisson-Nijenhuis structures, bihamiltonian systems and dynamical r-matrices. In [34], Liu and I proved that classical dynamical r-matrices are connected with a special class of Lie bialgebroids, called coboundary Lie bialgebroids [30, 50]. Using this result, we obtained a new method of classifying dynamical r-matrices of simple Lie algebras $\mathfrak{g}$, and established an explicit connection between the work of Etingof and Varchenko, that of Karolinsky on the classification of Lagrangian subalgebras and that of Lu on Poisson homogeneous spaces.

In [36, 29], L.-C. Li and I applied the idea of Lie algebroids to the study of integrable systems. In particular, we introduced dynamical Lie algebroids, which provide a general framework for studying integrable systems admitting the so called r-matrix formalism (depending on a dynamical parameter). As an application, we introduced spin Calogero-Moser systems associated with root systems of simple Lie algebras and gave the associated Lax representations (with spectral parameter) and fundamental Poisson bracket relations. Via Poisson reduction and gauge transformations, we obtained a new class of integrable models, called integrable spin Calogero-Moser systems. For Lie algebras of A_n-type, this new class of integrable systems includes the usual Calogero-Moser systems as subsystems.

In [40], I established an explicit correspondence between various geometric structures on a vector bundle with some well-known algebraic structures such as Gerstenhaber algebras and BV-algebras. As an application, I established an explicit connection between the Koszul-Brylinski operator and the modular class of a Poisson manifold, and proved that Poisson homology is isomorphic to Poisson cohomology for unimodular Poisson structures.

Some earlier work on Poisson geometry. In [60, 62, 63], I developed the theory of Morita equivalence for Poisson manifolds, and used it as a tool to study some geometric structures of Poisson manifolds such as symplectic realizations. In [57], I found a method of computing explicitly the Poisson cohomology for a certain class of regular Poisson manifolds. In [58], Liu and I undertook a general study of quadratic Poisson structures. In particular, we classified all three dimensional quadratic Poisson structures up to a Poisson diffeomorphism. I developed the theory of non-commutative Poisson structures in [11], with the aim to deal with badly behaved quotient spaces of Hamiltonian group actions. In the paper [59], Weinstein and I constructed geometric solutions to the set-theoretic quantum Yang-Baxter equation, solving a problem of Drinfeld. I introduced hyper-Lie Poisson structures in [49], and found an explicit example of a hyper-Lie Poisson structure, where the coadjoint orbits of $\mathfrak{sl}(2,\mathbb{C})$ are realized as hypersymplectic leaves. In [47], I generalized the flux homomorphism of symplectic manifolds to Poisson manifolds. By using it, I found a geometric description of a Lie group integrating the Poisson Lie algebra $C^\infty(P)$ for any compact Poisson manifold P.

In [26], I introduced the notion of Dirac submanifolds of Poisson manifolds. They play a similar role to Poisson manifolds as symplectic submanifolds of a symplectic manifold. These submanifolds provide a useful tool of constructing new examples of Poisson manifolds. They include the Poisson structure on Stokes matrices as discovered by Dubrovin and many others.

Quantization. In [38], Nistor, Weinstein and I developed the theory of pseudodifferential operators on a class of groupoids that generalizes differentiable groupoids to allow manifolds with corners. We also studied the symbol calculus. As applications, we gave a new proof of the Poincaré-Birkhoff-Witt theorem for Lie algebroids and a concrete quantization of the Lie-Poisson structure on the dual A^* of a Lie algebroid. In [44], I proved a number of results related to Fedosov $*$-products. Foremost is that every Fedosov $*$-product is a Vey $*$-product. As a consequence, I obtained a simpler proof of a classical theorem of Lichnerowicz that every $*$-product is equivalent to a Vey $*$-product. Another result came from deformation quantization of Hamiltonian G-spaces by introducing the notion of quantum momentum maps. In particular, I proved that the Poisson dual pair $\mathfrak{g}^* \xleftarrow{J} M \xrightarrow{pr} M/G$ of Weinstein can be quantized to a pair of mutual commutants in the $*$-algebra $C^\infty(M)[[\hbar]]$ using a quantum momentum map. In [42], Weinstein and I found a concrete intrinsic description of the characteristic class of an arbitrary star-product on a symplectic manifold, which recovers some earlier results of Deligne and De Wilde concerning the obstruction to the existence of a quantum Liouville operator.

I also applied the theory of deformation quantization to the study of quantization of dynamical r-matrices. In [32], I studied general properties of triangular dynamical r-matrices from the viewpoint of Poisson geometry. In particular, I proved that a triangular dynamical r-matrix always gives rise to a regular Poisson manifold. By using star-products, I proved that non-degenerate triangular dynamical r-matrices are quantizable, and the quantization is classified by a relative Lie algebra cohomology. This quantization method was also generalized to the so called splittable triangular dynamical r-matrices, which include all the known examples of triangular dynamical r-matrices. Finally, we arrive a conjecture that the quantization for an arbitrary triangular dynamical r-matrix is classified by the formal neighborhood of this r-matrix in the moduli space of triangular dynamical r-matrices. The dynamical r-matrix cohomology is introduced as a tool to understand such a moduli space.

In [31], I generalized this method to dynamical r-matrices over a nonabelian base. As an application, I obtained a geometric construction of a non-degenerate triangular dynamical r-matrix from a fat reductive decomposition $\mathfrak{g} = \mathfrak{h} \oplus \mathfrak{m}$ by using symplectic fibrations. By quantizing the corresponding Poisson manifold, I derived a new equation: the generalized quantum dynamical Yang-Baxter equation. Solutions to this equation was subsequently found by Enriquez and Etingof.

To unify various quantum objects such as star-products and quantum groups, I developed the theory of quantum universal enveloping algebroids, or quantum groupoids [35, 39, 43], which are quantizations of Lie bialgebroids. We extended to this general context some basic constructions in quantum groups such as the twist construction. In particular, I proved that a star-product is equivalent to a twist of the standard cocommutative Hopf algebroid on the algebra of differential operators. I also formulated a conjecture on the existence of a quantization for any Lie bialgebroid, and proved this conjecture for the special case of regular triangular Lie bialgebroids. As an application of this theory, I introduced dynamical quantum groupoids $\mathcal{D} \otimes_\hbar U_\hbar(\mathfrak{g})$, which give an interpretation of the quantum dynamical Yang-Baxter equation in terms of Hopf algebroids.

Noncommutative Geometry. Another research topic which I have been working on extensively is on differential stacks. Grothendiek introduced stacks initially to give geometric meaning to higher non-commutative cohomology classes. This is also the context in which *gerbes* first appeared in Giraud's work. However most of the work on stacks so far remains algebraic, though there is increasing evidence that differentiable stacks will find many useful applications. One example is orbifolds. In algebraic geometry, these correspond to Deligne-Mumford stacks. In differential geometry, orbifolds or V-manifolds have been studied for many years using local charts. Recently, it has been realized that viewing orbifolds as a very special kind of Lie groupoid is very useful. Behrend and I [6] established a dictionary between differentiable stacks and

Lie groupoids. Roughly speaking, differential stacks are Lie groupoids *up to Morita equivalence*. In particular we established a one-one correspondence between S^1-gerbes over a differentiable stack and Morita equivalence classes of groupoid S^1-central extensions. Applying Giraud's theory of non-abelian cohomology, we studied Dixmier-Douady classes for S^1-gerbes over differentiable stacks, which are in general integer third cohomology classes. We obtained a higher analogue of prequantization theorem of Kostant-Weil in the context of differentiable stacks [6]. Our work was motivated by string theory in which "gerbes with connections" appear naturally. For manifolds, there has been extensive work on this subject by Brylinski, Hitchin, Murray and many others. There is also interesting work on equivariant gerbes by Meinrenken, Gawedzki-Neis and others. These endeavors make the foundations of gerbes over differentiable stacks a very important subject.

Application of our work on differential stacks and gerbes includes geometric quantization and twisted K-theory. The K-theory of a topological space M twisted by a torsion class in $H^3(M, Z)$ was first studied by Donovan-Karoubi in the early 1970s. In the 1980s, using the theory of C^*-algebras, Rosenberg introduced K-theory twisted by a general element of $H^3(M, Z)$. More recently, twisted K-theory has enjoyed renewed vigor due to the discovery of its close connection with string theory. In particular, Atiyah-Segal rediscovered Rosenberg's twisted K-theory using the Fredholm type picture. In the mean-time, there has emerged a great deal of interest in twisted K-theory of other types, in particular, that of orbifolds and twisted equivariant K-theory. For instance, Adem–Ruan introduced a version of twisted K-theory of an orbifold by a discrete torsion element. Freed–Hopkins–Teleman showed that the twisted equivariant K-theory groups of a semi-simple compact Lie group is isomorphic to the Verlinde algebra. As an application of the general theory of S^1-gerbes over stacks developed by Behrend and myself, with Tu and Laurent-Gengoux [24], I took an important step by developing the twisted K-theory for differentiable stacks $\mathfrak{X}$, where the twisted class is given by a class in $H^3(\mathfrak{X}, \mathbb{Z})$, when the stack is proper. Our theory contains two important special cases: orbifold twisted K-theory and equivariant twisted K-theory. The latter was recently also introduced independently by Atiyah-Segal using a different method. The advantage of our approach is that it provides a uniform framework for studying various twisted K-theories. It also enables one to use various techniques in C^*-algebras and non-commutative geometry to attack problems in twisted K-theories. For instance, in [12] Tu and I proved that, under certain mild condition, twisted equivariant K-theory groups admit a ring structure, which was conjectured from string theory. In [21] we also studied the Chern-Connes character map for twisted K-theory of orbifolds. We introduced the twisted cohomology $H_c^*(\mathfrak{X}, \alpha)$ and proved that the Chern-Connes character map establishes an isomorphism between the twisted K-groups $K_\alpha^*(\mathfrak{X}) \otimes \mathbb{C}$ and the twisted cohomology $H_c^*(\mathfrak{X}, \alpha)$.

In a recent book in Astérisque [5], Behrend, Ginot, Noohi and I established the general machinery of string topology for differentiable stacks. This machinery allows us to treat free loops in stacks and hidden loops on an equal footing. In particular, we worked out a good notion of a free loop stack, and of a mapping stack $\mathrm{Map}(Y, \mathfrak{X})$, where Y is a compact space and $\mathfrak{X}$ a topological stack, which is functorial both in $\mathfrak{X}$ and Y and behaves well enough with respect to pushouts. We developed a bivariant (in the sense of Fulton and MacPherson) theory for topological stacks: it gives us a flexible theory of Gysin maps which are automatically compatible with pullback, pushforward and products. We proved that the homology of the free loop stack of an oriented stack is a BV-algebra and a Frobenius algebra, and the homology of hidden loops is a Frobenius algebra. We also established a relation between the string product of almost complex orbifolds and the so called twisted orbifold intersection pairing.

Non-abelian gerbes are gaining importance in various fields of mathematical physics, and in particular in quantization theory due to the work of Kashiwara and Kontesivch on algebroids of stacks. With Laurent-Gengoux and Stienon, in [14], I studied differential geometry of non-abelian differential gerbes over stacks using the theory of Lie groupoids. In particular, we introduced G-central extensions of groupoids (a notion generalizing groupoid S^1-central extensions), which correspond to G-bound gerbes, i.e. gerbes with trivial band. We also study connections on differential G-gerbes over stacks. In particular, we developed a cohomology theory that encodes the obstruction to the existence of connections and curvings for G-gerbes over stacks. Recently, Breen and Laurent-Gengoux proved that our theory of connections and curvings is essentially equivalent to that of Breen and Messing. According to Dedecker and Breen, a G-gerbe over a stack is equivalent to a 2-group $(G \to \mathrm{Aut}(G))$-principal bundle. Thus it is natural to study cohomology of (the classifying space) of a 2-group. In [11], Ginot and I studied the cohomology of (strict) Lie 2-groups. We obtained an explicit Bott-Shulman-type map in the case of a Lie 2-group corresponding to the crossed module $A \to 1$. The cohomology of the Lie 2-groups corresponding to the universal crossed modules $G \to \mathrm{Aut}(G)$ and $G \to \mathrm{Aut}^+(G)$ is the abutment of a spectral sequence involving the cohomology of $GL(n, \mathbb{Z})$ and $SL(n, \mathbb{Z})$. When the dimension of the center of G is less than 3, we explicitly compute these cohomology groups.

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