Higher Structure in Geometry and Physics

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Paul Baum

Algebra Deformation and Topological Representation theory

Let G be a locally compact Hausdorff second countable topological group. Examples are Lie groups, p-adic groups, adelic groups and discrete groups. What does it mean to understand the representation theory of such a group at a topological level — and what does this have to do with algebra deformation? This talk will take up these topics : especially for Lie groups and p-adic groups. For Lie groups the convolution algebra whose simple modules form the support of the Plancherel measure deforms to an algebra whose representation theory is readily understood. This was observed by G.Mackey, E.Wigner etc some decades ago. More recently P.Baum and A.Connes conjectured that the deformation is K-theory preserving. This conjecture has now been proved (G.Kasparov, V.Lafforgue, A.Wassermann). For a p-adic group G the analogous conjecture (A.-M. Aubert, P.Baum, R.Plymen) for the smooth representation theory of G uses the Hecke algebra of G and periodic cyclic homology instead of K-theory.

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Lawrence Breen

Connexions and Cohomology

I will discuss some relations between the concepts of connection (and curvature), and appropriate non-abelian cohomology classes.

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Giovanni Felder

Elliptic gamma functions, gerbes and triptic curves

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Kenji Fukaya

Homological algebra for open Gromov-Witten theory of higher genus

I this talk I want to explain some ideas to formulate algebraic structures useful to describe the moduli space of pseudo-holomorphic map from bordered Riemann surface of arbitrary genus bounding given Lagrangian submanifolds.

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Ezra Getzler

Descent for L-infinity algebras and n-groupoids

We describe the interaction between two notions of descent: the descent data for a cosimplicial n-groupoid (itself a special case of the total space, in the sense of Bousfield and Kan, of a cosimplicial space), and the total complex of a cosimplicial L-infinity algebra. The first of these notions of descent is as in Duskin (in "An outline of a theory of higher-dimensional descent"); the second is new.

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Tony Giaquinto

45 Years of Algebraic Deformations

Deformation theory as the study of continuous families of structures of some type can be traced back at least to Riemann, who in his 1857 treatise on abelian functions calculated the number of moduli of manifolds of complex dimension one. A century later, Froelicher-Nijenjuis and Kodaira-Spencer made fundamental insights into the development of the theory of deformations of complex manifolds of higher dimensions. In the early 1960s, Gerstenhaber introduced the theory of algebra deformations in a famous series of papers in which the intimate connections between deformations and cohomology were exposed. Throughout the years, the Gerstenhaber philosophy of deformation and cohomology has been adapted to countless types of algebraic structures and it has always played a central in deformation quantization, from its birth in the 1970s, to the celebrated proof by Kontsevich that every Poisson manifold can be quantized, and beyond.

This talk will be a sampling of the rich history of algebraic deformation and cohomology theories from their inception to the current day. The process will take us through such topics as deformations and cohomology of commutative algebras and the Hodge decomposition (and relation to the analytic Hodge decomposition), bialgebra cohomology and deformations, quantum groups, preferred deformations, explicit formulae, deformations of algebra diagrams (presheaves), etc. The recent notion of "variations" of algebras will also be presented as a way to partially resolve the paradox of how cohomologically trivial (and therefore rigid) algebras can appear in parametrized families. The first quantum Weyl algebra the function algebra on a four-punctured sphere are examples of this phenomenon.

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Simone Gutt

Quantization and special connections

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Johannes Huebschmann

Origins and Breadth of the Theory of Higher Homotopies

Higher homotopies are nowadays playing a prominent role in mathematics as well as in certain branches of theoretical physics. The purpose of the talk is to recall some of the connections between the past and the present developments. Higher homotopies were isolated within algebraic topology at least since the 1940's. Prompted by the failure of the Alexander-Whitney multiplication of cocycles to be commutative, Steenrod developed certain operations which measure this failure in a coherent manner. Dold and Lashof extended Milnor's classifying space construction to associative H-spaces, and a careful examination of this extension led Stasheff to the discovery of A_n-spaces and A_∞-spaces as a notion which controls the failure of associativity in a coherent way so that the classifying space construction can still be pushed through.

Algebraic versions of higher homotopies have, as we all know, led Kontsevich eventually to the proof of the formality conjecture. Homological perturbation theory (HPT), in a simple form first isolated by Eilenberg and Mac Lane in the early 1950's, has nowadays become a standard tool to handle algebraic incarnations of higher homotopies. A basic observation is that higher homotopy structures behave much better relative to homotopy than strict structures, and HPT enables one to exploit this observation in various concrete situations. For example, certain invariants can be calculated which are otherwise intractable.

Higher homotopies abound but they are rarely recognized explicitly and their significance is hardly understood; at times, their appearance might at first glance even come as a surprise, for example in the Kodaira-Spencer approach to deformations of complex manifolds or in the theory of foliations.

I finally plan to show how an exploration of suitable homotopies in a particular geometric situation leads to a construction of line bundles on certain moduli spaces and of a geometric object which, given a compact Lie group, depends functorially on a chosen invariant inner product on the Lie algebra and represents the cohomology class given by the Cartan 3-form. This geometric object may thus be viewed as an alternative to the familiar equivariant gerbe representing the first Pontrjagin class of the classifying space.

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Hiroshige Kajiura

Homological perturbation theory and homological mirror symmetry: the $\mathbb{R}^2$ case

As an explicit example of an A_∞-structure associated to geometry, we construct a Fukaya A_∞-category of lines (Lagrangian) in $\mathbb{R}^2$. This is motivated by homological mirror symmetry of (two-)tori, where $\mathbb{R}^2$ is the covering space of a two-torus. The key idea is to apply homological perturbation theory for A_∞-structures to Morse theory as discussed by Kontsevich and Soibelman. Technically, this is interesting since this can give a way to avoid the problem of transversality of Lagrangians in the Fukaya category.

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Mikhail Kapranov

Free Lie algebroids and the space of paths

For a smooth manifold X, we have the infinite-dimensional Lie groupoid of unparametrized paths in X. We define what should be its Lie algebroid. It is a version of the free Lie algebra construction in the context of Lie algebroids. Sections of this algebroids can be seen as "noncommutative vector fields" which act in every bundle with connection. One has the enveloping algebra of "noncommutative differential operators". The corresponding concept of integral operators involves integration over the space of paths, via a noncommutative version of the Fourier transform.

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Bernhard Keller

Categorification of acyclic cluster algebras

We will recall the combinatorial construction of the cluster algebra associated with a quiver without loops or 2-cycles following Fomin and Zelevinsky. We will then report on the categorical interpretation of this algebra in the case of a quiver without oriented cycles (for example a Dynkin quiver). The main results presented at the end of the talk were obtained in joint work with Philippe Caldero

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Maxim Kontsevich

On Calabi-Yau categories and their moduli

Boundary conditions in any topological conformal theory form a triangulated A-infinity category with Calabi-Yau property. I will give an overview of Calabi-Yau triangulated categories and present some new results: 1) geometric examples, 2) degeneration of Hodge-de Rham spectral sequence, abstract Tian-Todorov lemma amd TQFT, 3) categorical crepant resolutions and conifold transitions, 3) local Calabi-Yau spaces, relation with cluster mutations.

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Yvette Kosmann-Schwarzbach

Gleaning in the (modular) fields: topics in Lie algebroid theory

Emphasizing the role of Gerstenhaber algebras and of higher derived brackets in the theory of Lie algebroids, we shall prove that the various Lie algebroids defined in the recent literature can all be defined in terms of Poisson and pre-symplectic functions in the sense of Roytenberg and Terashima. We shall study the modular classes of those Lie algebroids, as well as the relative modular classes of general morphisms of Lie algebroids (joint work with Camille Laurent-Gengoux and Alan Weinstein), and prove a simple formula which permits an explicit computation of the modular class of a regular Poisson or twisted Poisson structure on a Lie algebroid, in particular in the case of the triangular r-matrices on Lie algebras associated to Frobenius structures (joint work with Milen Yakimov).

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Janko Latschev

Symplectic field theory and string topology

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Jean-Louis Loday

Beyond Hopf algebras

In the theory of Hopf algebras there is a structure theorem which is very useful. It says that, in characteristic zero, a connected cocommutative Hopf algebra is cofree as a coalgebra and is, as an algebra, isomorphic to the universal enveloping algebra of a Lie algebra. This structure theorem is essentially equivalent to the union of the Poincare-Birkhoff-Witt theorem with the Cartier-Milnor-Moore theorem. It involves three types of algebras, that is three operads: Com for the coalgebra structure, As for the algebra structure, and Lie for the structure of the primitive part. The purpose of this talk is to show that there are numerous other examples of this form, many of them already in the literature. We consider generalized bialgebras of type $C^c - A$, where C refers to the coalgebra structure and A to the algebra structure. We give elementary conditions on the $C^c - A$ type so that there is an analogous structure theorem for $C^c - A$-bialgebras.

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Sergei Merkulov

Quantization of strongly homotopy Lie bialgebras

Using theory of props we prove a formality theorem associated with universal quantizations of (strongly homotopy) Lie bialgebras.

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Pierre Schapira

Finiteness and duality on complex symplectic manifolds

A complex symplectic manifold (stx, ω) is endowed with the cor-algebroid stack W[stx] of deformation quantization. Here, cor $=$ W[rmpt] is subfield of $\mathbb{C}[[\mathrm{opb}\,\tau, \tau]$. We consider the triangulated category $\mathrm{RD}^{\mathrm{Rb}}_{\mathrm{gd}}(\mathrm{W}[\mathrm{stx}])$ consisting of objects whit good cohomology (roughly speaking, coherent modules endowed with a good filtration) and its subcategory $\mathrm{RD}^{\mathrm{Rb}}_{\mathrm{gdc}}(\mathrm{W}[\mathrm{stx}])$ of objects with compact supports.

We prove that, under a natural properness condition, the composition $\mathrm{shk}_2 \circ \mathrm{shk}_1$ of two good kernels $\mathrm{shk}_i \in \mathrm{RD}^{\mathrm{Rb}}_{\mathrm{gd}}(\mathrm{W}[\mathrm{stx}_{i+1} \times \mathrm{stx}^a_i])$ $(i = 1, 2)$ is a good kernel and that this composition commutes with duality. (Here, stx^a is the manifold stx endowed with the symplectic form $-\omega$.)

As a particular case, we obtain that the triangulated category $\mathrm{RD}^{\mathrm{Rb}}_{\mathrm{gdc}}(\mathrm{W}[\mathrm{stx}])$ is Ext-finite over the field cor and admits a Serre functor, namely the shift $[\dim \mathrm{stx}]$. We also explain how the data of an object shk of $\mathrm{RD}^{\mathrm{Rb}}_{\mathrm{gd}}(\mathrm{W}[\mathrm{stx} \times \mathrm{stx}^a \times \mathrm{stx}^a])$ satisfying suitable properties allows us to endow the category $\mathrm{RD}^{\mathrm{Rb}}_{\mathrm{gdc}}(\mathrm{W}[\mathrm{stx}])$ of a structure of tensor category.

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Daniel Sternheimer

The deformation philosophy, quantization of space time and baryogenesis

We start with a brief survey of the notions of deformation in physics and in mathematics, in particular Gerstenhaber's seminal works, and present the deformation philosophy in physics promoted by Flato since the 70's, examplified by deformation quantization and its manifold avatars, including the more recent quantization of space-time. Deforming Minkowski space-time to anti de Sitter has significant physical consequences (e.g. singleton physics). We shall present an ongoing program in which anti de Sitter would be quantized in some regions, speculating that this could explain baryogenesis in a universe in constant expansion.

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Dennis Sullivan

Model Algebras for Various Theories with Systems of Moduli Spaces

There are several instances where geometric theories lead to invariants defined by the homological information carried by a system of chains living on a system of moduli spaces.These systems are sometimes defined by nonlinear PDEs as in Donaldson theory and Symplectic Topology, by transversality as in String Topology, or they are just there as in the configuration spaces of 3D manifolds leading to invariants of knots and three manifolds.A common feature of all these examples is that the boundary of one moduli space in the system can be described by some recipe involving earlier moduli spaces in the system for a natural partial order. Model algebra is a construct adapted to fit these various moduli space recipes, on the one hand, and to be amenable to the expected patterns of homotopical algebra on the other.

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Charles Torossian

Bi-quantization and applications for symmetric spaces

In a joint work with A. Cattaneo we investigate the case of general symmetric pair with Kontsevich methods.

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Boris Tsygan

On the Gauss-Manin connection in cyclic homology

We define Getzler's Gauss-Manin connection in cyclic homology at the level of chains and discuss its relation to noncommutative differential calculus.

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Alan Weinstein

$S^1/\mathbb{Z}$: quantum group or groupoid?

When the singular quotient space $S^1/\mathbb{Z}$ is modeled by the symplectic 2-torus (classical limit of the quantum torus), the group structure becomes a groupoid structure. This fact seems to reflect the close reationship between hopfish algebras (algebras with a coproduct given by a bimodule) and Hopf algebroids.