

Math 429, Quiz 9.

Your name:

1. Calculate the fundamental group of annulus:

$$A = \{z \in \mathbb{C} \mid 1 \leq |z| \leq 2\}.$$

Solution: *The space A is homeomorphic to $S^1 \times [0, 1]$, thus*

$$\pi_1(A) = \pi_1(S^1) \times \pi_1([0, 1]) = \mathbb{Z}.$$

2. Let X be a path connected topological space and

$$f : X \rightarrow S^1, \quad g : S^1 \rightarrow X$$

be two continuous maps. Assume

$$f \circ g(s) = s \quad \text{for all } s \in S^1.$$

Prove that X is not simply connected.

Solution: *Assume X is s.c., then there is a null-homotopy $g_t : S^1 \rightarrow X$, $g_0 = g$. Then $f \circ g_t$ gives a null-homotopy of identity map $S^1 \rightarrow S^1$, a contradiction.*