This book emerged from the course in fractal geometry and dynamical systems with emphasis on chaotic dynamics that I taught in the Fall semester of 2008 as part of the MASS program at Penn State University.

Both fractal geometry and dynamical systems have a long history of development that is associated with many great names, Poincaré, Kolmogorov, Smale and Cantor, Hausdorff, Besicovich to name a few. These two areas interact with each other since many dynamical systems (even some very simple ones) often produce fractal sets that are a source of irregular "chaotic" motions in the system.

A unifying factor for merging dynamical systems with fractal geometry is self-similarity. On the one hand, self-similarity, along with complicated geometric structure, is a crucial feature of fractal sets. On the other hand, it is related to various symmetries in dynamical systems (e.g., rescaling of time or space). This is extremely important in applications as symmetry is an attribute of many physical laws, which govern the processes described by dynamical systems.

Numerous examples of scaling and self-similarity resulting in appearance of fractals and chaotic motions are explored in the fascinating book by Schroeder []. Motivated by this book, I designed and taught a course – for a group of undergraduate and graduate students majoring in various areas of science – whose goal was to describe necessary mathematical tools to study many of the examples in Schroeder's book. An expanded and modified version of this previous course has become the course for MASS students that I mentioned above.

The book is aimed at undergraduate students and requires only standard knowledge in analysis and differential equations but the topics covered do not fall into the traditional undergraduate curriculum and may be demanding. To help the reader cope with this we give formal definition of notions that are not part of the standard undergraduate curriculum (e.g., of topology, metric space and measure) and we briefly discuss them. Furthermore, many crucial new concepts are introduced through examples so that the reader can get some motivation for their necessity as well as some intuition of their meaning and role.

The focus of the book is on ideas rather then on complicated techniques. Consequently, the proofs of some statements, which require rather technical arguments, are restricted to some particular cases that, while allowing for simpler methods, still capture all the essential elements of the general case. Moreover, to help the reader get a broader view on the subject we included some results whose proofs go far beyond the scope of the book. Naturally, these proofs are omitted.

Currently, there are some text books for undergraduate students that introduce the reader to the dynamical system theory (see for example, []) and to fractal geometry (see for example, []) but neither of them presents a systematic study of their interplay and connections to the theory of chaos. This book is meant to cover this gap.

Chapter 1 of the book starts with a discussion of the principle three-cord of dynamics, fractals and chaos. Here our core example is introduced – a one-dimensional linear Markov map whose biggest invariant set is a fractal and whose "typical" trajectories are chaotic. Although this map is governed by a very simple rule, it exhibits all the principle features of dynamics that are important for our purposes.

After being immersed into the interplay between dynamics and fractal geometry the reader is invited to a more systematic study of dimension theory and its connections to dynamical systems, which are presented in Chapters 2, 3 and 4. Here the reader finds among other things rigorous definitions of various dimensions and description of their basic properties; various methods for computing dimensions of sets, most importantly of Cantor sets; and relations between dimension and some other characteristics of dynamics.

Chapters 5 through 9 are dedicated to two "real-life" examples of dynamical systems – the FitzHugh-Nagumo model and the Lorenz system where the former describes the propagation of signal through the axon of a neuron cell and the latter models the behavior of fluid between two plates heated to different temperatures. While the underlining mechanism in the FitzHugh-Nagumo model is a map of the plane, the Lorenz system is a system of differential equations in the three-dimensional space. This allows the reader to observe various phenomena associated with dynamical systems with discrete time (maps) as well as with continuous time (flows).

An important feature of these two examples is that each system depends on some parameters (of which one is naturally selected to be the leading parameter) so that the behavior of the system varies (bifurcates) when the leading parameter changes. Thus the reader becomes familiar in a somewhat natural way with various types of behavior emerging when the parameter changes including homoclinic orbits, Smale's horseshoes and "strange" (or "chaotic") attractors.

Let me say a few words on how the book was writen. My coauthor, Vaughn Climenhaga (who at the time of writing the book was the fourth year graduate student) was the TA for the MASS course that I taught. He was responsible for taking and writing up notes. He did this amazingly fast (usually within one or two days after the lecture) so that the students could have them in "real time". The notes embellished with many interesting details, examples and some stories that he added on his own were so professionally written that, with few exceptions, I had to do only some minor editing before they were posted on the web. These notes have become the ground material for the book. Turning them into the book required adding some new material, restructuring and editing. Vaughn's participation in this process was at least an equal share but he also produced all the pictures, the TeX source of the book, etc. I do not think that without him this book would have ever been written.

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