

MATH 220
Fall 2017
Quiz 9

Name: SOLUTION
Section Number: _____

Instructions: This is a closed-book quiz. Be sure to show **ALL** your work, as this is a partial credit quiz. Full credit will not be given for answers which are not accompanied by some justification.

Let

$$A = \begin{bmatrix} 1 & 2 & 1 & 0 \\ -3 & 7 & 2 & -1 \\ 0 & 2 & 0 & 0 \\ 3 & 6 & -4 & 1 \end{bmatrix}.$$

1. (4 points) Compute $\det(A)$.

Expand into cofactors along the third row

$$\begin{aligned} \det(A) &= (-2) \det \begin{bmatrix} 1 & 1 & 0 \\ -3 & 2 & -1 \\ 3 & -4 & 1 \end{bmatrix} \quad \text{expand along first row} \\ &= (-2) \left(\det \begin{bmatrix} 2 & -1 \\ -4 & 1 \end{bmatrix} - \det \begin{bmatrix} -3 & -1 \\ 3 & 1 \end{bmatrix} \right) \\ &= (-2) \left((2-4) - 0 \right) = 4 \end{aligned}$$

2. (6 points)

(a) Is A invertible? Explain your answer.

A is invertible since $\det A \neq 0$

(b) Compute $\det(A^T A^2)$.

$$\det(A^T A^2) = \det(A^T) \det(A)^2 = \det(A)^3 = 4^3 = 64$$

(c) Compute $\det(2(A^{-1})^3)$.

$$\begin{aligned} \det(2(A^{-1})^3) &= 2^4 \det(A^{-1})^3 \quad (\text{since } A^{-1} \text{ is } 4 \times 4 \text{ matrix}) \\ &= \frac{2^4}{\det(A)^3} = \frac{2^4}{4^3} = \frac{1}{4} \end{aligned}$$