MATH 220
Fall 2017
**Quiz 12**

Name: SOLUTION

Section Number: _____

**Instructions:** This is a closed-book quiz. Be sure to show **ALL** your work, as this is a partial credit quiz. Full credit will not be given for answers which are not accompanied by some justification.

In all problems

$$\mathbf{y} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \quad \text{and} \quad \mathbf{v} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}.$$

1. (4 points) Compute the orthogonal projection of $\mathbf{y}$ onto the line through $\mathbf{v}$ and the origin.

$$\hat{y} = \left(\frac{\vec{y}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\right)\vec{v} = \frac{-1+2+0}{1+1+1}\vec{v} = \frac{1}{3}\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$$

2. (3 points) Write $\mathbf{y}$ as the sum of a vector in Span($\mathbf{v}$) and a vector orthogonal to $\mathbf{v}$.

$\vec{y} = \hat{y} + \vec{z}$ where

$$\vec{z} = \vec{y} - \hat{y} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} - \begin{bmatrix} -1/3 \\ 1/3 \\ 1/3 \end{bmatrix} = \begin{bmatrix} 4/3 \\ 5/3 \\ -1/3 \end{bmatrix}$$

so

$$\vec{y} = \begin{bmatrix} -1/3 \\ 1/3 \\ 1/3 \end{bmatrix} + \begin{bmatrix} 4/3 \\ 5/3 \\ -1/3 \end{bmatrix}$$

3. (3 points) Compute the shortest distance from $\mathbf{y}$ to this line through $\mathbf{v}$ and the origin.

$$\|\vec{z}\| = \sqrt{\left(\tfrac{4}{3}\right)^2 + \left(\tfrac{5}{3}\right)^2 + \left(\tfrac{1}{3}\right)^2} = \sqrt{\frac{16+25+1}{9}}$$

$$= \sqrt{\frac{42}{9}} = \sqrt{\frac{14}{3}}$$