MATH 220 Name: SOLUTION
Fall 2017
Quiz 11 Section Number: ____________

Instructions: This is a closed-book quiz. Be sure to show **ALL** your work, as this is a partial credit quiz. Full credit will not be given for answers which are not accompanied by some justification.

1. (10 points) Find matrices P and D which provide diagonalization of

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}.$$

If such a diagonalization is not possible, explain why.

First compute the characteristic polynomial

$$\det(A - \lambda I) = \det \begin{bmatrix} 2-\lambda & 3 \\ 4 & 1-\lambda \end{bmatrix} = (2-\lambda)(1-\lambda) - 3\cdot 4$$

$$= \lambda^2 - 3\lambda - 10 = (\lambda-5)(\lambda+2).$$

Thus the eigenvalues of A are 5 and -2.

For $\lambda = 5$:

$A - 5I = \begin{bmatrix} -3 & 3 \\ 4 & -4 \end{bmatrix}$. The equation $(A-5I)\vec{x} = \vec{0}$ amounts to $x_1 - x_2 = 0$, so $x_1 = x_2$ with x_2 free. The general solution is $x_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, and a basis vector for the eigenspace is $\vec{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

For $\lambda = -2$

$A + 2I = \begin{bmatrix} 4 & 3 \\ 4 & 3 \end{bmatrix}$. $(A+2I)\vec{x} = \vec{0}$ amounts to $4x_1 + 3x_2 = 0$, so general solution is $x_2 \begin{bmatrix} -3/4 \\ 1 \end{bmatrix}$, and a nice basis vector for the eigenspace is $v_2 = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$.

We conclude

$$\boxed{D = \begin{bmatrix} 5 & 0 \\ 0 & -2 \end{bmatrix}, \quad P = \begin{bmatrix} 1 & -3 \\ 1 & 4 \end{bmatrix}}$$