Solutions to Quiz 9

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PSU Math 220. Thursday, 4/8/8

Problem 1 *$T : \mathbf{R}^2 \mapsto \mathbf{R}^2$ is the $180°$ rotation about the origin. Equivalently, it is the reflection through the origin. Find all its eigenvectors.*

It is easy to notice that $T(\vec{x}) = -\vec{x} = (-1)\vec{x}$ for any $\vec{x}$ in $\mathbf{R}^2$, for this is exactly what the reflection through the origin does. Recall that $\vec{x} \neq \vec{0}$ is an eigenvector of T if there is such λ that $T(\vec{x}) = \lambda\vec{x}$. It is easy to find such a lambda: -1. So any nonzero $\vec{x}$ in $\mathbf{R}^2$ is an eigenvector of T, because it satisfies $T(\vec{x}) = (-1)\vec{x}$. ($\vec{0}$ is never an eigenvector by definition)

One can also start by finding the standard matrix for T. The standard matrix for T is $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$. If you don't remember how to find the standard matrix of a linear transformation (and for that matter, what is the standard matrix of a linear transformation), check the section *1.9* and the exercises therein. This is one of the very basic things in the course, and I would be surprized if you get a 'C' without knowing it.

From this matrix it is easy to find that the only eigenvalue is $\lambda = -1$. (The characteristic polynomial is $\lambda^2 + 2\lambda + 1$, so it has only one root: -1, of multiplicity 2.) The eigenspace corresponding to this eigenvalue is $Nul(A - (-1)I) = Nul(A + I)$. However, $A = -I$, so this is $Nul(0)$, the null space of the 2×2 zero matrix. This null space is simply $\mathbf{R}^2$. So the eigenspace, corresponding to $\lambda = -1$, is the whole of $\mathbf{R}^2$, and all nonzero vectors in this eigenspace are eigenvectors corresponding to the eigenvalue -1. Again, the answer is: all nonzero vectors in $\mathbf{R}^2$.

The answer $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ is wrong as there are infinitely many eigenvectors, not just two. The answer $\mathbf{R}^2$ is technically wrong since $\vec{0}$ is not an eigenvector. The answer $\lambda = -1$ does not address the question, as the question was to find all eigenvectors.

Exercise: consider the $90°$ rotation of $\mathbf{R}^3$ around the z-axis (choose either direction). Find all eigenvectors of this linear transformation.