

Answer to Mock Exam 1, MATH 251 (Feb. 22, 2006)

75 minutes

1. True or false?

- (a) The equation $y'(t) = y(t) + t$ is an autonomous ODE. (T) (F) False
- (b) The equation $u_{tt} - u_{xx} = 0$ is a nonlinear, 2nd order PDE. (T) (F) False
- (c) The equation $M(y) + N(x)y' = 0$ is exact. (T) (F) False
- (d) According to the Existence and Uniqueness Theorem, this initial value problem has a unique solution

$$\frac{dy}{dt} = y^{1/3} - \frac{y}{4}, \quad y(0) = 8.$$

(T) (F) True

2. Consider the first-order linear equation

$$ty' + (t - 2)y = \tan t$$

with initial condition $y(t_0) = y_0$. Existence and Uniqueness Theorem applies

- (a) at every point.
- (b) every where except $t_0 = 0, 2, \pi/2$.
- (c) for sure in the interval $0 < t_0 < 2$.
- (d) for sure in the interval $0 < t_0 < \pi/2$. *This is the right choice.*

3. A tank contains 100 gallons of water and 50 oz of salt. Water containing a salt concentration of $\frac{1}{4}(1 + \frac{1}{2} \sin t)$ oz/gal flows into the tank at a rate of 2 gal/min, and the mixture in the tank flows out at the same rate. Establish a differential equation for the amount of salt in the tank at any time. (Multiple choices of answers will be provided in the real exam.)

Answer: $Q'(t) = \frac{1}{2}(1 + \frac{1}{2} \sin t) - \frac{1}{50}Q$.

4. Find the critical points for the equation $y' = y(1 - y)(1 + y)$ and determine their stability (stable or unstable).

Answer: Critical points are $-1, 0, 1$. Stable points are $-1, 1$. Unstable point is 0 .

5. Use variable substitution to solve

$$ty'' + y' = 1, \quad (t > 0).$$

Simplify your final answer.

$$\text{Answer: } y = c_1 + c_2 \ln |t| + t.$$

6. Solve the initial-value problem

$$y' = 3t - ty, \quad y(1) = 0.$$

$$\text{Answer: } y = 3 - 3e^{\frac{1}{2} - \frac{t^2}{2}}.$$

7. Solve the initial-value problem

$$y' = (t + 1)y^2, \quad y(0) = 2.$$

$$\text{Answer: } y = \frac{2}{1 - t^2 - 2t}.$$

8. Solve the initial value problem

$$y'' + 4y' + 4y = 0, \quad y(0) = 0, \quad y'(0) = -1.$$

$$\text{Answer: } y = -te^{-2t}.$$