

**PLEASE WRITE CLEARLY** No calculator, text, notes, or homework.

Name:

## Mock Exam 1, MATH 251 (February 20, 2006)

75 minutes

### 1. (20 points) True or false?

(a) The equation  $y'(t) = y(t) + t$  is an autonomous ODE. (T) (F)

(b) The equation  $u_{tt} - u_{xx} = 0$  is a nonlinear, 2nd order PDE. (T) (F)

(c) The equation  $M(y) + N(x)y' = 0$  is exact. (T) (F)

(d) According to the Existence and Uniqueness Theorem, this initial value problem has a unique solution

$$\frac{dy}{dt} = y^{1/3} - \frac{y}{4}, \quad y(0) = 8.$$

(T) (F)

### 2. (10 points) Consider the first-order linear equation

$$ty' + (t - 2)y = \tan t$$

with initial condition  $y(t_0) = y_0$ . Existence and Uniqueness Theorem applies

(a) at every point.

(b) every where except  $t_0 = 0, 2, \pi/2$ .

(c) for sure in the interval  $0 < t_0 < 2$ .

(d) for sure in the interval  $0 < t_0 < \pi/2$ .

**3.** A tank contains 100 gallons of water and 50 oz of salt. Water containing a salt concentration of  $\frac{1}{4}(1 + \frac{1}{2} \sin t)$  oz/gal flows into the tank at a rate of 2 gal/min, and the mixture in the tank flows out at the same rate. Establish a differential equation for the amount of salt in the tank at any time. (Multiple choices of answers will be provided in the real exam.)

4. Find the critical points for the equation  $y' = y(1-y)(1+y)$  and determine their stability (stable or unstable).

5. Use variable substitution to solve

$$ty'' + y' = 1, \quad (t > 0).$$

Simplify your final answer.

6. Solve the initial-value problem

$$y' = 3t - ty, \quad y(1) = 0.$$

7. Solve the initial-value problem

$$y' = (t + 1)y^2, \quad y(0) = 2.$$

8. Solve the initial value problem

$$y'' + 4y' + 4y = 0, \quad y(0) = 0, \quad y'(0) = -1.$$