

Answer Key for Practice Exam for Exam II

1.

Answer: $y = c_1 e^{-t} + c_2 e^{-t/2} + t^2 - 6t + 14 - \frac{3}{10} \sin t - \frac{9}{10} \cos t$.

2.

(a) Answer: $u'' + 2u' + u = 0$.

(b) Answer: $u(1) = \frac{2}{e}$.

(c) The motion is critically damped and the $u(t)$ decays exponentially to the equilibrium position $u = 0$ without oscillating.

3.

Answer: $c_1 + c_2 t + c_3 e^t + c_4 e^{-t} + c_5 \sin t + c_6 \cos t$.

4.

Answer:

$$a_{n+2} = -\frac{(n^2 - 2n + 4)a_n}{2(n+1)(n+2)}, \quad \text{for } n \geq 2,$$

and $a_2 = -a_0$, and $a_3 = -a_1/4$.

5.

(a) Answer: $\mathcal{L}\{f(t)\}(s) = \int_0^\infty e^{-ts} f(t) dt$

(b) Answer: See Section 6.1.

(c) Answer: See Section 6.1.

6.

Answer: $y(t) = (e^{3t} + 4e^{-2t})/5$.

7.

Answer: $y(t) = (4 \cos(3t) + \cos(2t))/5$.

8.

Answer: $y(t) = 1 - \cos t + \sin t - u_{\pi/2}(t)(1 - \sin t)$.

9.

Answer: $y(t) = (\cos(2t) + u_{4\pi}(t) \sin(2t))/2$.

10.

Answer: $y(t) = \frac{1}{5} \cos t + \frac{2}{5} \sin t - \frac{1}{5} e^{-t} \cos t - \frac{3}{5} e^{-t} \sin t + u_{\pi/2}(t) e^{\frac{\pi}{2}-t} \sin(t - \frac{\pi}{2})$.