Answers to Quiz 2

Math 230. Wednesday, 9/20/6

Problem 1 (25%) *Check whether the plane of $\triangle ABC$ contains the point P.* $A = (1,0,0), B = (0,2,0), C = (1,1,1), P = (-2,1,-4)$.

$\overrightarrow{AB} = \langle -1,2,0\rangle \neq \overrightarrow{BA} = \langle 1,-2,0\rangle$. $\overrightarrow{AC} = \langle 0,1,1\rangle, \overrightarrow{AP} = \langle -3,1,-4\rangle$. One way is to check whether all four points A,B,C,P are coplanar. For that purpose one may check whether $\overrightarrow{AP}\cdot(\overrightarrow{AB}\times\overrightarrow{AC}) = \overrightarrow{0}$ or whether $\overrightarrow{CP}\cdot(\overrightarrow{AB}\times\overrightarrow{BC}) = \overrightarrow{0}$, but the fact that $\overrightarrow{OP}\cdot(\overrightarrow{AB}\times\overrightarrow{AC}) = 0$ with the points A, B, C and P being coplanar. These scalar triple products may be computed directly by a 3×3 determinant or finding the cross product and then the dot product.

Another way (which is very similar) is to find the equation of the plane. The normal vector is $\overrightarrow{n} = \overrightarrow{AB}\times\overrightarrow{AC} = \langle 2,1,-1\rangle$. Make sure to check that, for instance, $\overrightarrow{AB}\cdot\overrightarrow{n} = 0$. The equation of the plane is, therefore, $2x+y-z+d=0$ for some d. To find d, substitute the point A, which is definitely in the plane. This yields $d=2$, hence the equation of the plane is $2x+y-z-2=0$. (It is a good idea to substitute all three points A,B,C in this equation and to check whether they in fact satisfy it). Finally, substituting P: $2(-2)+1-(-4)-2 = -4+1+4-2 = -1 \neq 0$, hence P is not in the plane.

Note: if your solution makes no use of the coordinates of one of the points given, i.e. A or C, then it cannot be true. Note: any three points are always coplanar. More precisely, if three points do not lie on the same line then there is exactly one plane passing through all three of them.

Problem 2 (75%) *Find the symmetric equations of the line $\overrightarrow{r} = \langle 3,3,3\rangle + t\langle -1,-1,-1\rangle$.*

Immediately $x = 3-t$, $y = 3-t$, $z = 3-t$, but these three are the parametric equations of these line. Express t: $t = 3-x = 3-y = 3-z$. Thus the symmetric equations of the line are: $3-x = 3-y = 3-z$.