Chapter 8 - FORMULA SHEET

Integration by Parts Formula

$$\int udv = uv - \int vdu$$

Integrating Trigonometric Functions

Useful Formulae and Identities

1. Half Angle Identities: $sin^2x = \frac{1-cos(2x)}{2}, cos^2x = \frac{1+cos(2x)}{2}$
2. $sin(2x) = 2sin(x)cos(x), cos(2x) = cos^2(x) - sin^2(x)$
3. $sec^2(x) - tan^2(x) = 1.$
4. $sin(A)cos(B) = \frac{1}{2}[sin(A-B) + sin(A+B)]$
5. $sin(A)sin(B) = \frac{1}{2}[cos(A-B) - cos(A+B)]$
6. $cos(A)cos(B) = \frac{1}{2}[cos(A-B) + cos(A+B)]$

Important Antiderivatives

1. $\int tan(x)dx = ln|sec(x)| + C$
2. $\int sec(x)dx = ln|sec(x) + tan(x)| + C$
3. $\int sec^2(x)dx = tan(x) + C$
4. $\int sec(x)tan(x)dx = sec(x) + C$

Strategy for evaluating $\int sin^m(x)cos^n(x)dx$

1. If n is *odd*, save one cosine factor and express the remaining factors in terms of sine. Then substitute $u = sin(x)$.
2. If m is *odd*, save one sine factor and express the remaining factors in terms of cosine. Then substitute $u = cos(x)$.
3. If both m and n are *even*, use the half-angle identities.

Strategy for evaluating $\int tan^m(x)sec^n(x)dx$

1. If n is *even*, then save one $sec^2(x)$ factor and express the remaining factors in terms of $tan(x)$. Now substitute $u = tan(x)$.
2. If m is *odd*, then save one $sec(x)tan(x)$ factor and express the remaining factors in terms of $sec(x)$. Now substitute $u = sec(x)$.
3. For the other cases, there are no clear-cut guidelines.

Integrating Using Trigonometric Substitution

To evaluate the following 3 types of integrals, we must make the corresponding trig substitution.

1. $\sqrt{a^2-u^2}$ $\quad u = asin\theta, \quad du = acos\theta d\theta \quad \sqrt{a^2-u^2} = acos\theta$
2. $\sqrt{a^2+u^2}$ $\quad u = atan\theta, \quad du = asec^2\theta d\theta \quad \sqrt{a^2+u^2} = asec\theta$
3. $\sqrt{u^2-a^2}$ $\quad u = asec\theta, \quad du = asec\theta tan\theta \quad \sqrt{u^2-a^2} = atan\theta$