SECTIONS 7.7 and 8.8 - FORMULA SHEET

7.7 - INDETERMINATE FORMS AND L'HOSPITAL'S RULE

Suppose f and g are differentiable.

- **TYPE 0/0** If $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = 0$ then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

- **TYPE** ∞/∞ If $\lim_{x\to a} f(x) = \pm\infty$ and $\lim_{x\to a} g(x) = \pm\infty$ then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

- **TYPE** $0 \cdot \infty$ If $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = \pm\infty$, then rewrite the product fg as a quotient:

$$fg = \frac{f}{1/g}, \qquad or \qquad fg = \frac{g}{1/g}$$

  to convert it to type 0/0 or type ∞/∞. Then use the L'Hospital's Rule to evaluate the limit.

- **TYPE** $\infty - \infty$ If $\lim_{x\to a} f(x) = \infty$ and $\lim_{x\to a} g(x) = \infty$, then convert the limit

$$lim_{x\to a}[f(x) - g(x)]$$

  into type 0/0 or ∞/∞ by using a common denominator, rationalization, or factoring out a common factor. Then use L'Hospital's rule to evaluate the limit.

- **TYPE** $0^0, \infty^0, 1^\infty$ Let $y = f(x)^{g(x)}$. Take ln of both sides, use properties of logarithms to simplify, find the limit L, and finally take e^L to get the answer.

NOTE THAT THESE ARE THE ONLY TYPES OF INDETERMINATE FORMS.

SECTION 8.8 IMPROPER INTEGRALS

- (a) $\int_a^\infty f(x)dx = \lim_{t\to\infty} \int_a^t f(x)dx$. The integral is called **convergent** if the limit exists and **divergent** if it does not exist.
- (b) $\int_{-\infty}^b f(x)dx = \lim_{t\to-\infty} \int_t^b f(x)dx$. The integral is called **convergent** if the limit exists and **divergent** if it does not exist.
- (c) $\int_\infty^\infty f(x)dx = \int_{-\infty}^a f(x)dx + \int_a^\infty f(x)dx$ for any real number a, provided both the integrals are convergent.
- (d) If f is continuous on $[a, b)$ but discontinuous at b, then $\int_a^b f(x)dx = \lim_{t\to b^-} \int_a^t f(x)dx$.
- (e) If f is continuous on $(a, b]$ but discontinuous at a, then $\int_a^b f(x)dx = \lim_{t\to a^+} \int_t^b f(x)dx$.
- (f) If f has a discontinuity at c, where $a < c < b$ then $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$, provided both the limits on the right hand side exist and are finite.

EXAMPLE: The integral $\int_a^\infty \frac{1}{x^p}dx$ is convergent if $p > 1$ and divergent if $p \leq 1$.