

CHAPTER 11 - FORMULA SHEET

11.1. Parametric Curve

1. The **parametric equations of a curve** are the pair:

$$x = f(t), \quad y = g(t), \quad a \leq t \leq b.$$

Here, a and b could be $\pm\infty$.

2. For smooth curves, these equations can be **differentiated** with respect to t . By using the chain rule for first derivatives, we get that

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}.$$

In other words, the **slope of the tangent line** at any point (x, y) on C is the ratio of the vertical to horizontal velocity. The **equation of the tangent line** at the point $t = a$ is given by

$$y - y(a) = \frac{dy}{dx}(x - x(a)).$$

3. The **second derivative** is given by

$$y'' = \frac{d^2y}{dx^2} = \frac{\left(\frac{dy'}{dt}\right)}{\left(\frac{dx}{dt}\right)},$$

where $y' = \frac{dy}{dx}$ is the first derivative.

The curve is **concave upwards wherever $y'' > 0$** .

11.2. Polar Coordinate Systems

1. The polar coordinate system consists of an **origin**, or **pole** O and the **polar axis**, which is usually chosen to be the horizontal axis.
2. The **Polar Coordinates of a point** P are specified by two numbers, (r, θ) . Here r is the **distance** OP from the origin to the point, and θ is the **angle measure**, in radians, between the polar axis and the line OP .
3. Angles are measured in the counter-clockwise direction and **negative values of r** correspond to reflecting through the origin from (r, θ) to $(-r, \theta) = (r, \theta + \pi)$.
4. To convert from (x, y) to (r, θ) , use

$$x = r\cos\theta, \quad y = r\sin\theta.$$

5. To convert from (r, θ) to (x, y) , use

$$r = \sqrt{x^2 + y^2}, \quad \theta = \tan^{-1}\left(\frac{y}{x}\right).$$

You must choose the value of θ based on the appropriate quadrant in which the point (x, y) lies. This is because the inverse tangent function is not single-valued everywhere.

11.3. Polar Curves

Curves in Polar Coordinate systems are called Polar Curves, which can be written as $r = f(\theta)$ or, equivalently, as $F(r, \theta) = 0$. The following steps can be used for graphing polar curves:

1. First try to convert to x and y coordinates, by multiplying by r if necessary and/or a suitable trig substitution. If the resulting equation is simple to graph, then that is the graph.
2. If Step 1 fails, then make a graph/table of the appropriate trigonometric function and trace out the curves.

11.4. Area Formulae

We can use the idea that the area of a circular sector of radius r and central angle θ is $\frac{1}{2}r^2\theta$ to prove the following area formula:

The area of the region enclosed by the polar curve $r = f(\theta)$ between the rays $\theta = a$ to $\theta = b$ is given by

$$\int_a^b \frac{1}{2} f(\theta)^2 d\theta.$$