FINAL EXAM - FORMULA SHEET

NEWTON'S METHOD

Newton's Formula for the solution to $f(x) = 0$ with initial approximation x_0 (such that $f'(x_0) \neq 0$) is given by

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Start with $x_0 = a$, find x_1 by using $n = 1$ in this formula, then use this value of x_1 to find x_2 using $n = 2$, and so on.

THE THEORY OF INTEGRATION

You must memorize by yourselves all these formulas if you want to pass calculus! You can do it if you practice everyday!

1. **Definition of Integrals**

   The **area** A of the region S that lies under the graph of the **continuous** function f is the same as the **definite integral**, which is defined as the **limit of the Riemann sum.**

   $$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

   Here we have divided $[a, b]$ into n subintervals of width $\Delta(x) = (b - a)/n$ and $a = x_0 \leq x_1 \leq x_2 ... \leq x_n = b$ are the endpoints of the ith subinterval $[x_{i-1}, x_i]$, while the x_i^* are the **sample points**, which could be **left endpoints, right endpoints, or midpoints.**

2. $\int_a^c f(x)dx + \int_c^b f(x)dx = \int_a^b f(x)dx$, if $a \leq c \leq b$.

3. $\int_b^a f(x)dx = -\int_a^b f(x)dx$.

4. **Powers** $\int x^n dx = \frac{x^{n+1}}{n+1}$, $n \neq -1$.

5. **Linearity** $\int(kf(x) + g(x))dx = k\int f(x)dx + \int g(x)dx$; for example: $\int (x^2 + 3x + 1)dx) = x^3/3 + 3x^2/2 + x + C$, where C is the constant of integration.

6. **Substitution** Let $u = g(x)$ be a differentiable function whose range is an interval I and f is continuous on I. Then

   $$\int f(g(x))g'(x)dx = \int f(u)du$$

   For **Definite Integrals,** remember to change the limits accordingly. Note the limits on the integral on the right are limits for u and **not for**x. Also remember to **check for discontinuities (zero on denominator** of integrand):

   If g' is continuous on $[a, b]$ and f is continuous on the range of $u = g(x)$ then

   $$\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$$

7. **Integrals of Symmetric Functions** Suppose f is continuous on $[-a, a]$.
   - If f is an even function (symmetric about y-axis), then $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$.
   - If f is an odd function (symmetric about the origin), then $\int_{-a}^{a} f(x)dx = 0$.

8. **Elementary Functions**
   - $\int sin(x)dx = -cos(x) + C$
   - $\int cos(x)dx = sin(x) + C$
   - $\int sec^2(x)dx = tan(x) + C$
   - $\int sec(x)tan(x)dx = sec(x) + C$
   - $\int csc(x)cot(x)dx = -csc(x) + C$
   - $\int csc^2(x)dx = -cot(x)dx + C$

9. **Fundamental theorem of Calculus** Suppose f is continuous on $[a, b]$,
   - **PART 1** : If $g(x) = \int_{a}^{x} f(t)dt$, then $g'(x) = f(x)$.
   - **PART 2** : $\int_{a}^{b} f(x)dx = F(b) - F(a)$, where F is the antiderivative of f, that is, $F' = f$.

   NOTE: The **upper limit in Part 1 can be a function** $p(x)$ instead of just x, in which case the formula becomes:

   $$g(x) = \int_{a}^{p(x)} f(t)dt \Rightarrow g'(x) = f(p(x))p'(x)$$

   .

10. **Areas between curves** The area of the region bounded by the curves $y = f(x), y = g(x)$, and the lines $x = a$ and $x = b$, where f and g are continuous is $A = \int_{a}^{b} |f(x) - g(x)|dx$.

11. **Volumes** Let S be a solid that lies between $x = a$ and $x = b$. If the cross-sectional area of S in the plane P_x, through x and perpendicular to the x-axis (**vertical slices**), is $A(x)$ then the volume of S is

    $$V = \int_{a}^{b} A(x)dx$$

    Each slice will be either a closed disk or a washer. This is the formula to use when you rotate about a **horizontal line**. Similarly, to rotate about a vertical line, use horizontal slices and the formula $V = \int_{c}^{d} A(y)dy$.

12. **Volume by Cylindrical Shells**

    Here we use cylindrical shells instead of slices as cross-sections. The formula becomes

    $$V = \int_{a}^{b} 2\pi x f(x)dx$$

13. **The average value of a function f on [a,b]** is

    $$f_{av} = \frac{1}{b-a}\int_{a}^{b} f(x)dx$$

14. **Mean Value Theorem for Integrals** If f is continuous on $[a, b]$, then there exists a number c in $[a, b]$ such that

    $$\int_{a}^{b} f(x)dx = f(c)(b - a)$$