1. Find two positive numbers whose product is 144 and whose sum is a minimum.
   a. 12, 12
   b. 2, 72
   c. 4, 36

2. Consider the following problem: A farmer with 760 ft of fencing wants to enclose a rectangular area and then divide it into four pens with fencing parallel to one side of the rectangle. What is the largest possible total area of the four pens?
   a. 14540 ft²
   b. 14440 ft²
   c. 14460 ft²
   d. 14429 ft²
   e. 14463 ft²
   f. 14439 ft²

3. If 2600 cm² of material is available to make a box with a square base and an open top, find the largest possible volume of the box.
   a. 12746 cm³
   b. 12857 cm³
   c. 12756 cm³
   d. 12777 cm³
   e. 12757 cm³
   f. 12780 cm³

4. A rectangular storage container with an open top is to have a volume of 7 m³. The length of its base is twice the width. Material for the base costs $15 per square meter. Material for the sides costs $5 per square meter. Find the cost of materials for the cheapest such container.
   a. $130.72
   b. $130.7
   c. $129.7
   d. $129.2
   e. $128.4
   f. $134.9
5 Find the point on the line \( y = 8x + 6 \) that is closest to the origin.

a. \( \left( \frac{-48}{64}, \frac{7}{64} \right) \)

b. \( \left( \frac{-47}{65}, \frac{6}{65} \right) \)

c. \( \left( \frac{-50}{65}, \frac{7}{65} \right) \)

d. \( \left( \frac{-48}{64}, \frac{6}{65} \right) \)

e. \( \left( \frac{-48}{65}, \frac{8}{65} \right) \)

f. \( \left( \frac{-48}{65}, \frac{6}{65} \right) \)

6 Find the dimensions of the rectangle of largest area that can be inscribed in an equilateral triangle of side \( L = 3 \) cm if one side of the rectangle lies on the base of the triangle. Round the result to the nearest tenth.

a. 2.5 cm, 1.8 cm

b. 1.5 cm, 1.4 cm

c. 4.5 cm, 0.3 cm

d. 1 cm, 1.31 cm

e. 1.5 cm, 1.3 cm

f. 6.5 cm, 1.3 cm

7 A right circular cylinder is inscribed in a sphere of radius \( r = 3 \) cm. Find the largest possible surface area of such a cylinder. Round the result to the nearest hundredth.

a. 96.5 cm\(^2\)

b. 90.39 cm\(^2\)

c. 91.5 cm\(^2\)

d. 91.52 cm\(^2\)

e. 92 cm\(^2\)

f. 91.4 cm\(^2\)
8 A Norman window has the shape of a rectangle surmounted by a semicircle. (Thus the diameter of the semicircle is equal to the width of the rectangle.) If the perimeter of the window is 45 ft, find the dimensions of the window so that the greatest possible amount of light is admitted. Round the result to the nearest hundredth.

a. base = 12.58 ft, height = 7.3 ft
b. base = 12.6 ft, height = 6.3 ft
c. base = 13.6 ft, height = 6.27 ft
d. base = 12.6 ft, height = 5.8 ft
e. base = 12.71 ft, height = 6.3 ft
f. base = 12.7 ft, height = 6.5 ft

9 A piece of wire 19 m long is cut into two pieces. One piece is bent into a square and the other is bent into an equilateral triangle. How should the wire be cut for the square so that the total area enclosed is a minimum? Round the result to the nearest hundredth.

a. 9.26 m
b. 7.16 m
c. 19 m
d. 8.31 m
e. 0 m
f. 8.26 m

10 A fence 5 ft tall runs parallel to a tall building at a distance of 4 ft from the building. What is the length of the shortest ladder that will reach from the ground over the fence to the wall of the building? Round the result to the nearest hundredth.

a. 12.90 ft
b. 11.60 ft
c. 12.69 ft
d. 13.70 ft
e. 14.73 ft
f. 12.70 ft
A conical drinking cup is made from a circular piece of paper of radius \( R = 7 \text{ cm} \) by cutting out a sector and joining the edges \( CA \) and \( CB \). Find the maximum capacity of such a cup. Round the result to the nearest hundredth.

\[
\begin{align*}
A & \quad R \\
C & \quad B
\end{align*}
\]

a. 138.35 \( \text{cm}^3 \)
b. 138.27 \( \text{cm}^3 \)
c. 138.3 \( \text{cm}^3 \)
d. 137.24 \( \text{cm}^3 \)
e. 137.25 \( \text{cm}^3 \)
f. 138.25 \( \text{cm}^3 \)
12 A woman at a point A on the shore of a circular lake with radius 4 mi wants to arrive at the point C diametrically opposite on the other side of the lake in the shortest possible time. She can walk at the rate of 7 mi/h and row a boat at 3 mi/h. How should she proceed? (Find θ). Round the result, if necessary, to the nearest hundredth.

a. 0.65 radians
b. 0.49 radians
c. She should walk around the lake from point A to point C.
d. She should row from point A to point C radians
e. 0.44 radians
f. 0.77 radians

13 Find an equation of the line through the point (6, 12) that cuts off the least area from the first quadrant.

a. \( y = -3x + 24 \)
b. \( y = -2x + 24 \)
c. \( y = -2x + 25 \)
d. \( y = 2x + 24 \)
e. \( y = -3x + 25 \)
Consider the figure below, where $a = 8$, $b = 1$ and $l = 6$. How far from the point $A$ should the point $P$ be chosen on the line segment $AB$ so as to maximize the angle $\theta$? Round the result to the nearest hundredth.

- a. 3.83
- b. 4.14
- c. 3.17
- d. 4.43
- e. 3.13
- f. 3.3

A painting in an art gallery has height $h = 79$ cm and is hung so that its lower edge is a distance $d = 12$ cm above the eye of an observer (as seen in the figure below). How far from the wall should the observer stand to get the best view? (In other words, where should the observer stand so as to maximize the angle $\theta$ subtended at his eye by the painting?) Round the result to the nearest hundredth.

- a. 31.69 cm
- b. 35.57 cm
- c. 36.4 cm
- d. 33.05 cm
- e. 31.93 cm
- f. 33.14 cm
16 A steel pipe is being carried down a hallway 11 ft wide. At the end of the hall there is a right-angled turn into a narrower hallway 9 ft wide. What is the length of the longest pipe that can be carried horizontally around the corner? Round the result to the nearest hundredth.

\[
\text{Length of the longest pipe} = \sqrt{11^2 + 9^2}
\]

\[= \sqrt{121 + 81}
\]

\[= \sqrt{202}
\]

\[\approx 28.27 \text{ ft}
\]

17 Find the maximum area of a rectangle that can be circumscribed about a given rectangle with length \(L = 3\) and width \(W = 7\).

\[
\text{Maximum area} = 2 \times (L + W)
\]

\[= 2 \times (3 + 7)
\]

\[= 2 \times 10
\]

\[= 20
\]

\[\approx 20 \text{ sq ft}
\]
The graph of a function $f(x)$ is given below. Suppose that Newton's method is used to approximate the roots $r$ and $s$ of the equation $f(x) = 0$. The blue (1) and red (2) lines are the tangent lines corresponding to initial approximations for finding these roots. In the approximation of the value of $r$, what is $x_2$?

- a. $x_2 \approx 11.9$
- b. $x_2 \approx 11$
- c. $x_2 \approx 7$
- d. $x_2 \approx 6.1$

Suppose the line $y = 8x - 7$ is tangent to the curve $y = f(x)$ when $x = 1$. If Newton's method is used to locate a root of the equation $f(x) = 0$ and the initial approximation is $x_1 = 1$, find the second approximation $x_2$.

- a. $x_2 = \frac{7}{8}$
- b. $x_2 = \frac{9}{8}$
- c. $x_2 = -\frac{7}{8}$
20 The graph of a function is given. For which of the following initial approximations does Newton's method fail?

a. \( x_1 = 1 \)

b. \( x_1 = 8 \)

c. \( x_1 = 9 \)

d. \( x_1 = 2 \)

21 Use Newton's method with the specified initial approximation \( x_1 \) to find \( x_4 \), the fourth approximation to the root of the given equation. (Give your answer to five decimal places.)

\( x^3 + x^2 - 9 = 0 \), \( x_1 = 3 \)

a. \( x_4 = 6.06612 \)

b. \( x_4 = 0.05603 \)

c. \( x_4 = 1.85215 \)

d. \( x_4 = 1.79612 \)

22 Use Newton's method with the specified initial approximation \( x_1 \) to find \( x_3 \), the third approximation to the root of the given equation. (Give your answer to four decimal places.)

\( x^4 - 29 = 0 \), \( x_1 = 4 \)

a. \( x_3 = 3.1133 \)

b. \( x_3 = 2.5752 \)

c. \( x_3 = 3.8186 \)

d. \( x_3 = 6.3052 \)
Use Newton's method to approximate the given number, correct to eight decimal places:

\[ \sqrt[3]{27} \]

a. 3.00000001
b. 2.99999999
c. 3.00000000
d. 2.99999998

Use Newton's method to approximate the root of

\[ x^4 + x - 8 = 0 \]

in the interval [1, 2], correct to six decimal places.

Use \( x_1 = 1.5 \) as the initial approximation.

a. \( x = 1.591094 \)
b. \( x = 1.591091 \)
c. \( x = 1.591095 \)
d. \( x = 1.591092 \)

Newton's method is used to approximate the least positive root of the equation:

\[ 10 \sin x = x \]

Using an initial approximation \( x_1 = 2.81 \), find the fifth approximation \( x_5 \).

a. \( x_5 = 2.852341 \)
b. \( x_5 = 2.85234 \)
c. \( x_5 = 2.852342 \)
d. \( x_5 = 2.852343 \)
26 Use Newton's method to find all the roots of the equation, correct to six decimal places.

\[ 2x^5 - 2x^4 - 21x^3 - 31x^2 - 296x - 120 = 0 \]

a. \( x = -7.162278 \)
b. \( x = -0.418861 \)
c. \( x = -3.581139 \)
d. \( x = 5 \)
e. \( x = -0.837722 \)

27 Use Newton's method to find all the roots of the equation, correct to four decimal places.

\[ x^4 - 50x^2 - 51 + \frac{15}{x^2 + 5} = \frac{x^2 + 20}{x^2 + 5} - 1 \]

a. \( x = -7.1414 \)
b. \( x = 7.1415 \)
c. \( x = 7.1404 \)
d. \( x = 7.1414 \)
e. \( x = -7.1404 \)

28 The following algorithm used by the ancient Babylonians to compute \( \sqrt{a} \):

\[ x_{n+1} = \frac{1}{2} \left( x_n + \frac{a}{x_n} \right) \]

(You can derive it by applying Newton's method to the equation \( x^2 - a = 0 \)).

Use this algorithm to compute \( \sqrt{2210} \), correct to six decimal places.

a. 47.010634
b. 47.010638
c. 47.010636
d. 47.010635
e. 47.010637
29 The following algorithm enables a computer to find reciprocals without actually dividing:

\[ x_{n+1} = 2x_n - ax_n^2 \]

(You can derive it by apply Newton's method to the equation \( \frac{1}{x} - a = 0 \).)

Use this algorithm to compute \( \frac{1}{1.725} \), correct to seven decimal places.

a. 0.5797100  
b. 0.5797105  
c. 0.5797098  
d. 0.5797102

30 The equation \( 2x^3 - 6x + 3 = 0 \) is given.

For some initial approximations, Newton's method doesn't work for finding the root of this equation. Which of the following are such values of \( x_1 \)?

a. \( x_1 = 4 \)  
b. \( x_1 = 8 \)  
c. \( x_1 = 1 \)  
d. \( x_1 = 9 \)  
e. \( x_1 = 1 \)

31 Find the initial approximation \( y_1 \) for which Newton's method succeeds when applied to the equation \( 4\sqrt[3]{y} = 0 \).

a. \( y_1 = 4 \)  
b. \( y_1 = 7 \)  
c. \( y_1 = -4 \)  
d. \( y_1 = 0 \)

32 A grain silo consists of a cylindrical main section, with height 38 ft, and a hemispherical roof. In order to achieve a total volume of 23000 ft\(^3\) (including the part inside the roof section), what would the radius of the silo have to be? Find the result and round to four decimal places.

a. \( r = 12.5644 \) ft  
b. \( r = 12.5657 \) ft  
c. \( r = 12.7030 \) ft  
d. \( r = 12.5595 \) ft
33 Find the most general antiderivative of the function:

\[ f(x) = 21x^2 - 4x + 10 \]

a. \[ F(x) = 21x^3 - 4x^2 + 10x + C \]

b. \[ F(x) = 7x^3 - 2x^2 + 10x + C \]

c. \[ F(x) = 35x^5 - 8x^4 + 10x + C \]

34 Find the most general antiderivative of the function:

\[ f(x) = \frac{1}{10}x^\frac{1}{4} \]

a. \[ F(x) = \frac{11}{10}x^\frac{5}{4} + C \]

b. \[ F(x) = \frac{9}{10}x^\frac{3}{4} + C \]

c. \[ F(x) = \frac{11}{10}x^\frac{5}{4} + C \]

35 Find the most general antiderivative of the function:

\[ f(x) = \frac{6}{x^7}, \quad x \neq 0 \]

a. \[ F(x) = -\frac{1}{8}x + C \]

b. \[ F(x) = -\frac{1}{6}x + C \]

c. \[ F(x) = \frac{1}{6}x + C \]
36 Find the most general antiderivative of the function:

\[ f(x) = 4 \cos x - 7 \sin x \]

a. \[ F(x) = 4 \sin(x) + 7 \cos(x) + C \]
b. \[ F(x) = 4 \sin(x) - 7 \cos(x) + C \]
c. \[ F(x) = -4 \sin(x) + 7 \cos(x) + C \]

37 Find \( f \):

\[ f''(x) = 18x + 24x^2 \]

a. \[ f(x) = 9x^3 + 4x^4 + Cx + D \]
b. \[ f(x) = 6x^3 + 8x^4 + Cx + D \]
c. \[ f(x) = 3x^3 + 2x^4 + Cx + D \]

38 Find \( f \):

\[ f''(x) = 81 \cos(9x) \]

a. \[ f(x) = y = 81 \cos(x) + Cx + D \]
b. \[ f(x) = -\cos(9x) + Cx + D \]
c. \[ f(x) = y = -\cos(9x) + Cx^2 + D \]

39 Find \( f \):

\[ f'(x) = 3 \cos(x) + 10 \sin(x) \]

\[ f(0) = 2 \]

a. \[ f(x) = 3\sin(x) + 10\cos(x) + 12 \]
b. \[ f(x) = -3\sin(x) - 10\cos(x) + 2 \]
c. \[ f(x) = 3\sin(x) - 10\cos(x) + 12 \]
40 Find \( f \):

\[ f''(x) = 12x \]

\[ f(2) = 27 \]
\[ f'(2) = 29 \]

a. \( f(x) = 3x^3 + 5x + 2 \)
b. \( f(x) = 3x^3 + 6x + 1 \)
c. \( f(x) = 2x^3 + 6x + 2 \)
d. \( f(x) = 2x^3 + 5x + 1 \)

41 Given that the graph of \( f \) passes through the point (3,13) and that the slope of its tangent line at \( (x,f(x)) \) is \( 6x - 5 \), find \( f(2) \).

a. 2 

b. 4 
c. 6 
d. 3 

42 The graph of a function \( y(x) \) is shown. Which graph is a possible graph for antiderivative of \( y(x) \)?

a. 3 
b. 1 
c. 2 

d. 3 

43 Evaluate \( f(x) = \sin(x^2) \), and tell whether its antiderivative \( F \) increasing or decreasing at the point \( x = -2 \) radians.

a. 0.757, decreasing 
b. 0.757, increasing 
c. 0.909, decreasing 
d. 0.909, increasing
A particle moves along a straight line with velocity function \( v(t) = 2 \sin(t) - 8 \cos(t) \) and its initial displacement is \( s(0) = 4 \). Find its position function.

a. \( s(t) = 6 - 2\cos(t) - 8\sin(t) \)
b. \( s(t) = 4 - 2\cos(t) - 8\sin(t) \)
c. \( s(t) = 2 - 2\cos(t) + 8\sin(t) \)
d. \( s(t) = 6 + 2\cos(t) - 8\sin(t) \)

A stone is dropped from the upper observation deck (the Space Deck) of a tower, 340m above the ground. Find the distance of the stone above ground level at time \( t \).

a. \( s(t) = 340 - 9.8t^2 \)
b. \( s(t) = 340 + 9.8t^2 \)
c. \( s(t) = 340 - 4.9t^2 \)
d. \( s(t) = 340 + 4.9t^2 \)

A stone was dropped off a cliff and hit the ground with a speed of 128 ft/s. What is the height of the cliff?

for g.

a. 512 ft
b. 16384 ft
c. 256 ft
d. 8192 ft

A company estimates that the marginal cost (in dollars per item) of producing items is \( 1.88 - 0.004x \). If the cost of producing one item is $433 find the cost of producing 100 items.

a. $168.00
b. $1198.24
c. $599.12
d. $1030.24

The linear density of a rod of length 4 m is given by \( \frac{1}{\sqrt{x}} \) in grams per centimeter, when it is measured in centimeters from one end of the rod. Find the mass of the rod.

a. 4 g
b. 40 g
c. 20 g
d. 2 g
49 What constant acceleration is required to increase the speed of a car from 20 ft/s to 30 ft/s in 5 s?

a. \( \frac{2}{s} \) ft/s

b. \( \frac{250}{s} \) ft/s

d. \( \frac{4}{s} \) ft/s

d. \( \frac{10}{s} \) ft/s

50 A car braked with a constant deceleration of 15 ft/s\(^2\), producing skid marks measuring 110 ft before coming to a stop. How fast was the car traveling when the brakes were first applied?

a. 28.72 ft / s

b. 40.62 ft / s

c. 114.89 ft / s

d. 57.45 ft / s

51 By reading values from the given graph of \( f \), use five rectangles to find a lower estimate for the area from \( x = 0 \) to \( x = 10 \) under the given graph of \( f \). Round your answer to the nearest tenth.

a. 27.7

b. 25.9

c. 30.5

d. 29.6

e. 26.8
Estimate to the hundredth the area from 1 to 5 under the graph of \( f(x) = \frac{5}{x} \) using four approximating rectangles and right endpoints.

- a. 5.78
- b. 4.98
- c. 6.02
- d. 5.80
- e. 8.22
- f. 6.42

Estimate the area from 0 to 5 under the graph of \( f(x) = 36 - x^2 \) using five approximating rectangles and right endpoints.

- a. 108
- b. 127
- c. 115
- d. 135
- e. 125
- f. 132

Approximate the area under the curve \( y = \sin x \) from 0 to \( \pi/3 \) using ten approximating rectangles of equal widths and right endpoints.

- a. 2.00
- b. 0.92
- c. 0.28
- d. 1.38
- e. 0.54
- f. 0.62

Approximate the area under the curve \( y = \frac{4}{x^2} \) from 1 to 2 using ten approximating rectangles of equal widths and right endpoints.

- a. 1.60
- b. 3.74
- c. 3.32
- d. 2.72
- e. 1.00
- f. 1.86
56 If \( f(x) = \frac{2}{\sqrt{x}}, \ 1 \leq x \leq 4 \), approximate the area under the curve using ten approximating rectangles of equal widths and left endpoints.

a. 3.55  
b. 2.63  
c. 3.79  
d. 4.57  
e. 4.51  
f. 6.15

57 If \( f(x) = \sin \sin x, \ 0 \leq x \leq \pi/2 \), approximate the area under the curve using ten approximating rectangles of equal widths and left endpoints.

a. 1.33  
b. 1.37  
c. 1.03  
d. 2.01  
e. 0.83  
f. 0.03

58 The speed of a runner increased steadily during the first three seconds of a race. Her speed at half-second intervals is given in the table. Find a lower estimate for the distance that she traveled during these three seconds.

<table>
<thead>
<tr>
<th>t (s)</th>
<th>0</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>v (ft/s)</td>
<td>0</td>
<td>5</td>
<td>9.4</td>
<td>13.7</td>
<td>15.4</td>
<td>15.8</td>
<td>16.4</td>
</tr>
</tbody>
</table>

a. 28.75  
b. 28.45  
c. 29.65  
d. 30.75  
e. 31.55  
f. 29.15
59 When we estimate distances from velocity data, it is sometimes necessary to use times $t_0$, $t_1$, $t_2$, ... that are not equally spaced. We can still estimate distances using the time periods $\Delta t = t_i - t_{i-1}$. For example, on May 7, 1992, the space shuttle Endeavor was launched on mission STS – 49, the purpose of which was to install a new perigee kick motor in an Intelsat communications satellite. The table, provided by NASA, gives the velocity data for the shuttle between liftoff and the jettisoning of the solid rocket boosters. Use this data to estimate an upper bound for the space shuttle Endeavor's height above Earth's surface 56 seconds after liftoff.

<table>
<thead>
<tr>
<th>Event</th>
<th>Time (s)</th>
<th>Velocity (ft/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Launch</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Begin roll maneuver</td>
<td>8</td>
<td>180</td>
</tr>
<tr>
<td>End roll maneuver</td>
<td>15</td>
<td>336</td>
</tr>
<tr>
<td>Throttle to 89%</td>
<td>20</td>
<td>458</td>
</tr>
<tr>
<td>Throttle to 67%</td>
<td>31</td>
<td>739</td>
</tr>
<tr>
<td>Throttle to 104%</td>
<td>55</td>
<td>1365</td>
</tr>
<tr>
<td>Maximum dynamic pressure</td>
<td>56</td>
<td>1414</td>
</tr>
<tr>
<td>Solid rocket booster separation</td>
<td>124</td>
<td>4046</td>
</tr>
</tbody>
</table>

- a. 48385
- b. 49378
- c. 48198
- d. 48599
- e. 49196
- f. 49323

60 The velocity graph of a braking car is shown. Use it to estimate to the nearest foot the distance traveled by the car while the brakes are applied. Use a left sum with $n = 7$.

- a. 21
- b. 12
- c. 14
- d. 16
- e. 22
- f. 18
61 The velocity graph of a car accelerating from rest to a speed of 9 km/h over a period of 10 seconds is shown. Estimate to the nearest integer the distance traveled during this period. Use a right sum with \( n = 10 \).

\[
\begin{align*}
   y &= \tan x, \quad 0 \leq x \leq \frac{\pi}{11} \\
   y &= \tan x, \quad 0 \leq x \leq \frac{\pi}{12} \\
   y &= \tan x, \quad 0 \leq x \leq \frac{\pi}{14} \\
   y &= \tan x, \quad 0 \leq x \leq \frac{\pi}{1} \\
   y &= \tan x, \quad 0 \leq x \leq \frac{\pi}{5} \\
   y &= \tan x, \quad 0 \leq x \leq \frac{\pi}{4}
\end{align*}
\]
63. Find an expression for the area from 2 to 10 under the curve \( y = x^5 \) as a limit.

\[
\begin{align*}
\text{a. } & \lim_{n \to \infty} \sum_{i=1}^{n} \left( 2 + \frac{11i}{n} \right)^5 \frac{9}{n} \\
\text{b. } & \lim_{n \to \infty} \sum_{i=1}^{n} \left( 2 + \frac{10i}{n} \right)^5 \frac{7}{n} \\
\text{c. } & \lim_{n \to \infty} \sum_{i=1}^{n} \left( 2 + \frac{9i}{n} \right)^5 \frac{10}{n} \\
\text{d. } & \lim_{n \to \infty} \sum_{i=1}^{n} \left( 2 + \frac{10i}{n} \right)^5 \frac{10}{n} \\
\text{e. } & \lim_{n \to \infty} \sum_{i=1}^{n} \left( 2 + \frac{9i}{n} \right)^5 \frac{9}{n} \\
\text{f. } & \lim_{n \to \infty} \sum_{i=1}^{n} \left( 2 + \frac{8i}{n} \right)^5 \frac{8}{n}
\end{align*}
\]

64. Use a computer algebra system to find the area from 0 to 4 under the curve \( y = x^6 \).

\[
\begin{align*}
\text{a. } & 2341.53 \\
\text{b. } & 2342.54 \\
\text{c. } & 2336.45 \\
\text{d. } & 2340.57 \\
\text{e. } & 2337.88 \\
\text{f. } & 2349.19
\end{align*}
\]

65. Use a computer algebra system to find the exact area of the region from 0 to 5 under the graph of \( y = e^{-x} \).

\[
\begin{align*}
\text{a. } & 0.64 \\
\text{b. } & 0.49 \\
\text{c. } & 1.14 \\
\text{d. } & 0.99 \\
\text{e. } & 0.87 \\
\text{f. } & 0.88
\end{align*}
\]
Use a computer algebra system to find the area of the region from 0 to $\frac{\pi}{3}$ under the cosine curve $y = \cos x$.

a. 0.74  

b. 0.18  

c. 1.14  

d. 1.59  

e. 0.94  

f. 0.87

Evaluate the Riemann sum for $f(x) = 10 - x^2$, $0 \leq x \leq 2$, with four subintervals, taking the sample points to be right endpoints.

a. 16.25  

b. 162.5  

c. 6.5

If $f(y) = \sqrt{y} - 10$, $1 \leq y \leq 6$, find the Riemann sum with $n = 5$ correct to 3 decimal places, taking the sample points to be midpoints.

a. -9.789816  

b. -40.856759  

c. -81.713519
A table of values of an increasing function $f(x)$ is shown. Use the table to find an upper estimate of \[ \int_{0}^{25} f(x) \, dx. \]

<table>
<thead>
<tr>
<th>$x$</th>
<th>$f(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>43</td>
</tr>
<tr>
<td>5</td>
<td>37</td>
</tr>
<tr>
<td>10</td>
<td>22</td>
</tr>
<tr>
<td>15</td>
<td>7</td>
</tr>
<tr>
<td>20</td>
<td>7</td>
</tr>
<tr>
<td>25</td>
<td>32</td>
</tr>
</tbody>
</table>

a. 405  
b. 510  
c. 135  
d. 162

The table gives the values of a function obtained from an experiment. Use the values to estimate \[ \int_{0}^{6} f(z) \, dz \] using three equal subintervals with left endpoints.

<table>
<thead>
<tr>
<th>$z$</th>
<th>$f(z)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>9.8</td>
</tr>
<tr>
<td>1</td>
<td>8.9</td>
</tr>
<tr>
<td>2</td>
<td>7.7</td>
</tr>
<tr>
<td>3</td>
<td>6.1</td>
</tr>
<tr>
<td>4</td>
<td>4.1</td>
</tr>
<tr>
<td>5</td>
<td>6.8</td>
</tr>
<tr>
<td>6</td>
<td>10.3</td>
</tr>
</tbody>
</table>

a. 36  
b. 43.2  
c. 15.4
71 Use the Midpoint Rule with \( n = 10 \) to approximate the integral.

\[
\int_{0}^{2} \sqrt{10 + t^2} \, dt
\]

a. 2.438679
b. 3.509618
c. 14.038474

72 Use the Midpoint Rule with \( n = 5 \) to approximate the integral.

\[
\int_{0}^{10} 7 \sin \sqrt{w} \, dw
\]

a. 16.735
b. 45.250
c. 31.494

73 Express the limit as a definite integral on the given interval.

\[
\lim_{x \to \infty} \sum_{i=1}^{n} 8 w_i \sin w_i \Delta w, \quad [8,10]
\]

a. \( \int_{8}^{10} 8wdw \)
b. \( \int_{8}^{10} 8w \sin w \, dw \)
c. \( \int_{0}^{10} 8w \sin w \, dw \)

74 Express the limit as a definite integral on the given interval.

\[
\lim_{x \to \infty} \sum_{i=1}^{n} \left[ 5 z_i^2 - 15 z_i \right] \Delta z, \quad [3,13]
\]

a. \( \int_{3}^{13} \left( 5 z^2 - 15 z \right) \, dz \)
b. \( \int_{3}^{13} \left( 5 z^2 + 15 z \right) \, dz \)
c. \( \int_{5}^{15} \left( 3 z^2 - 13 z \right) \, dz \)
75 Express the integral as a limit of sums. Then evaluate the limit.

\[ \int_{0}^{\frac{\pi}{2}} \sin 13x \, dx \]

a. \( \frac{2}{13} \)
b. \( \frac{\pi}{13} \)
c. \( \frac{1}{13} \)

76 Evaluate the integral by interpreting it in terms of areas.

\[ \int_{1}^{3} (1 + 5x) \, dx \]

a. 28
b. 22
c. 12

77 Evaluate the integral by interpreting it in terms of areas.

\[ \int_{-\frac{2}{1}}^{2} \sqrt{4 - x^2} \, dx \]

a. \(8\pi\)
b. \(2\pi\)
c. \(4\pi\)

78 Evaluate the integral by interpreting it in terms of areas.

\[ \int_{-4}^{0} \left(1 + \sqrt{16 - x^2}\right) \, dx \]

a. \(\frac{4\pi}{4}\)
b. \(\frac{4\pi}{4} + 4\)
c. \(8\pi + 4\)
79 Evaluate the integral by interpreting it in terms of areas.

\[
\int_{-1}^{3} (2 - x) \, dx
\]

a. 4
b. 5
c. -8

80 Given that \[ \int_{5}^{7} f(x) \, dx = \frac{2}{67} \], find \[ \int_{7}^{5} \sqrt{t} \, dt \].

a. \( -\frac{2}{67} \)
b. \( \frac{67}{2} \)

81 Express the sum as a single integral in the form \[ \int_{a}^{b} f(z) \, dz \].

\[ \int_{1}^{8} f(z) \, dz + \int_{8}^{13} f(z) \, dz \]

a. \( \int_{8}^{13} f(z) \, dz \)
b. \( \int_{1}^{8} f(z) \, dz \)
c. \( \int_{1}^{8} f(z) \, dz \)

82 If \[ \int_{2}^{14} f(x) \, dx = 3.1 \] and \[ \int_{9}^{14} f(x) \, dx = 0.6 \], find \[ \int_{2}^{9} f(x) \, dx \].

a. 2.5
b. 3.7
c. -2.5
83 Find $g'(x)$ by evaluating the integral using Part 2 of the Fundamental Theorem and then differentiating.

$$g(x) = \int_{\pi}^{x} (7 + \cos(t)) \, dt$$

a. \( \frac{dg(x)}{dx} = 7x + \sin(x) \)

b. \( \frac{dg(x)}{dx} = - \sin(x) \)

c. \( \frac{dg(x)}{dx} = 7 + \cos(x) \)

84 Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

$$g(x) = \int_{1}^{x} \sqrt{8 + 4t} \, dt$$

a. \( \frac{dg(x)}{dx} = \frac{4}{2\sqrt{8 + 4x}} \)

b. \( \frac{dg(x)}{dx} = \sqrt{7 + 4x} \)

c. \( \frac{dg(x)}{dx} = \sqrt{8 + 4x} \)

85 Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

$$g(x) = \int_{6}^{x} \sin(t) \, dt$$

a. \( \frac{dg(x)}{dx} = 5x^5 \sin(x) \)

b. \( \frac{dg(x)}{dx} = 6x^6 \sin(x) \)

c. \( \frac{dg(x)}{dx} = \frac{x^7}{7} \cos(x) \)
Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

\[ g(x) = \int_{x}^{4} 4\tan(t) \, dt \]

a. \( \frac{dg(x)}{dx} = -4\tan(x) \)
b. \( \frac{dg(x)}{dx} = 4\tan(x) \)
c. \( \frac{dg(x)}{dx} = 4\tan(4) \)

Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

\[ g(x) = \int_{x}^{2} 8\sqrt{1 + t^8} \, dt \]

a. \( \frac{dg(x)}{dx} = 16x\sqrt{1 + x^{16}} \)
b. \( \frac{dg(x)}{dx} = 8\sqrt{1 + x^{16}} \)
c. \( \frac{dg(x)}{dx} = 16x\sqrt{1 + x^{8}} \)

Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

\[ g(x) = \int_{x}^{\sqrt{5}} \frac{\cos(t)}{t} \, dt \]

a. \( \frac{dg(x)}{dx} = 2.5 \frac{\cos(\sqrt{x})}{x} \)
b. \( \frac{dg(x)}{dx} = 5\cos(\sqrt{x}) \)
c. \( \frac{dg(x)}{dx} = 2.5 \frac{\cos(\sqrt{x})}{\sqrt{x}} \)

Evaluate the integral.

\[ \int_{0}^{3} \left( 3 + 6y - y^2 \right) \, dy \]

a. 45
b. 36
c. -27
d. 27
Evaluate the integral. \( \int_{0}^{3} x^5 \, dx \)

a. \( \frac{8}{5} \)

b. \( \frac{8}{3} \)

c. \( \frac{5}{8} \)

d. \( \frac{3}{8} \)

Evaluate the integral. \( \int_{0}^{1} \sqrt{x} \, dx \)

a. 1

b. 0.5

c. \( \frac{2}{3} \)

d. \( \frac{1}{3} \)

Evaluate the integral. \( \int_{0}^{8\pi} \cos \vartheta \, d\vartheta \)

a. 0

b. 1

c. \(-1\)

d. 2

Evaluate the integral. \( \int_{0}^{2\pi} \sin t \, dt \)

a. 1.732

b. 0.433

c. \(-0.866\)

d. \(-1.732\)

e. 0.866
94 Find the area of the region that lies beneath the given curve.

\[ y = \sin x, \ 0 \leq x \leq \frac{\pi}{4} \]

a. - 0.293  

b. 1.707  

c. - 1.707  

d. 0.293

95 Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

\[ g(x) = \int_{x^2}^{x^2 + 9} \frac{t^2 + 9}{t^2 - 9} \, dt \]

a. \( \frac{dg(x)}{dx} = -5 \frac{x^2 + 9}{x^2 - 9} + 10 \frac{x^2 + 9}{x^2 - 9} \)

b. \( \frac{dg(x)}{dx} = -5 \frac{25x^2 + 9}{25x^2 - 9} + 10 \frac{100x^2 + 9}{100x^2 - 9} \)

c. \( \frac{dg(x)}{dx} = \frac{25x^2 + 9}{25x^2 - 9} + \frac{100x^2 + 9}{100x^2 - 9} \)

96 Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

\[ g(x) = \int_{\cos(x)}^{4x} \cos(t^2) \, dt \]

a. \( \frac{dg(x)}{dx} = 4\cos((4x)^2) - \sin(x)\cos(\cos^2(x)) \)

b. \( \frac{dg(x)}{dx} = \cos((4x)^2) + \sin(x)\cos(x) \)

c. \( \frac{dg(x)}{dx} = 4\cos((4x)^2) + \sin(x)\cos(\cos^2(x)) \)

97 If \( F(x) = \int_{1}^{x} f(t) \, dt \), where \( f(t) = \int_{1}^{t} \sqrt{7 + \frac{u^4}{u}} \, du \), find \( F''(x) \).

a. \( \sqrt{263} \)

b. \( 2\sqrt{263} \)

c. \( \frac{\sqrt{263}}{2} \)
Find the interval on which the curve

\[ F(x) = \int_0^x \frac{1}{2 + 5t} \, dt \]

is concave upward.

a. \((- \infty, + \infty)\)

b. \((- \frac{2}{5}, \infty)\)

c. \((- \infty, - \frac{2}{5})\)

Let

\[ f(x) = \begin{cases} 
0 & \text{if } x < 0 \\
x & \text{if } 0 \leq x \leq 6 \\
12 - x & \text{if } 6 < x \leq 12 \\
0 & \text{if } x > 12 
\end{cases} \]

and

\[ g(x) = \frac{x}{0} \int f(t) \, dt. \]

Find an expression for \(g(x)\) when \(6 < x < 12\).

a. \(g(x) = 12x - \frac{1}{2} x^2 - 18\)

b. \(g(x) = 12x - \frac{1}{2} x^2 - 36\)

c. \(g(x) = 12x - \frac{1}{2} x^2\)

Find a function \(f(x)\) such that

\[ 2 + \int_a^{x} \frac{f(t)}{t^2} \, dt = 8\sqrt{x} \]

for \(x > 0\) and some \(a\).

a. \(f(x) = -\frac{5}{8x^{\frac{3}{2}}}\)

b. \(f(x) = 4x^{\frac{3}{2}}\)

c. \(f(x) = \frac{8}{3} x^{\frac{3}{2}} - 2x\)
A high-tech company purchases a new computing system whose initial value is $V$. The system will depreciate at the rate $f = f(t)$ and will incur maintenance costs at the rate $g = g(t)$, where $t$ is the time measured in months. The company wants to determine the optimal time to replace the system.

Suppose that

$$f(t) = \begin{cases} \frac{V}{20} - \frac{V}{800}t & \text{if } 0 < t \leq 42 \\ 0 & \text{if } t > 42 \end{cases}$$

Determine the length of time for the total depreciation $D(t) = \int_0^t f(s) \, ds$ to equal the initial value $V$.

a. $T = 800$

b. $T = 20$

c. $T = 40$

102 Find the general indefinite integral.

$$\int x \left( 6 + \frac{7}{x^5} \right) \, dx$$

a. $3x^2 + 7x^7 + C$

b. $3x^2 + \frac{7}{6} x^6 + C$

c. $6x + \frac{7}{7} x^7 + C$

d. $3x^2 + \frac{7}{7} x^7 + C$
103 Find the general indefinite integral.

\[ \int (9 - t) \left( 10 + t^2 \right) dt \]

a. \[90t - 5t^2 + 3t^3 + \frac{t^4}{4} + C\]
b. \[19t - 5t^2 + 3t^3 - \frac{t^4}{4} + C\]
c. \[90t - 5t^2 - 3t^3 - \frac{t^4}{4} + C\]
d. \[90t - 5t^2 + 3t^3 - \frac{t^4}{4} + C\]
e. \[90t - 5t^2 + 3t^3 - t^4 + C\]
f. \[90t + 3t^2 + 5t^3 - \frac{t^4}{4} + C\]

104 Find the general indefinite integral.

\[ \int \frac{\sin 12t}{\sin 6t} dt \]

a. \[-\frac{\cos 6t}{3} + C\]
b. \[\frac{\sin 6t}{6} + C\]
c. \[\frac{\cos 6t}{3} + C\]
d. \[-\frac{\sin 6t}{3} + C\]
e. \[\frac{\sin 6t}{3} + C\]

d. Evaluate the integral. \[\int \frac{9x^2 + 4}{4\sqrt{x}} dx\]

a. 430
b. 215
c. 92.4
d. 150
106
Evaluate the integral. 
\[ \int_{0}^{\frac{\pi}{6}} \frac{4 + \cos^2 \theta}{\cos^2 \theta} \, d\theta \]

a. 2.309  
b. 7.452  
c. 2.833  
d. 2.148

107
Evaluate the integral. 
\[ \int_{-4}^{5} 4x - x^2 \, dx \]

a. 251.67  
b. 166.33  
c. 145.00  
d. 66.33  
e. 151.67

108
The area of the region that lies to the right of the y-axis and to the left of the parabola \( x = 3y - y^2 \) (the shaded region in the figure) is given by the integral \( \int_{0}^{3} (3y - y^2) \, dy \).

Find the area of the region.

a. 2.25  
b. \( \frac{27}{6} \)  
c. \( \frac{9}{6} \)  
d. 27
109 If \( h' \) is a child's rate of growth in pounds per year, which of the following expressions represents the increase in the child's weight (in pounds) between the years 8 and 10?

a. \( \int_{8}^{10} h'(t) \, dt \)

b. \( h'(10) - h'(8) \)

110 Evaluate the integral. \( \int_{-1}^{2} x^4 \, dx \)

a. 33

b. \( \frac{15}{4} \)

c. \( \frac{9}{4} \)

d. \( \frac{31}{5} \)

e. \( \frac{33}{5} \)

111 The velocity function (in meters per second) is given for a particle moving along a line. Find the distance traveled by the particle during the given time interval.

\( v(t) = 5t - 3, \, 0 \leq t \leq 2 \)

a. 1m

b. 4m

c. 16m

d. 18.5m

112 The acceleration function (in m/s\(^2\)) and the initial velocity are given for a particle moving along a line. Find the velocity at time \( t \) and the distance traveled during the given time interval.

\( a(t) = t + 5, \, v(0) = 5, \, 0 \leq t \leq 10 \)

a. \( v(t) = \frac{t^2}{2} + 5 \text{ m/s}, \, 48\frac{1}{3} \text{ m} \)

b. \( v(t) = \frac{t^2}{2} + 5t \text{ m/s}, \, 456\frac{2}{3} \text{ m} \)

c. \( v(t) = \frac{t^2}{2} + 5t + 5 \text{ m/s}, \, 461\frac{2}{3} \text{ m} \)

d. \( v(t) = \frac{t^2}{2} + 5t + 5 \text{ m/s}, \, 466\frac{2}{3} \text{ m} \)
113 An animal population is increasing at a rate of \( 21 + 56t \) per year (where \( t \) is measured in years). By how much does the animal population increase between the fourth and tenth years?

a. 2226  

b. 4830  

c. 2478  

d. 2373  

114 The velocity of a car was read from its speedometer at ten-second intervals and recorded in the table. Use the Midpoint Rule to estimate the distance traveled by the car.

<table>
<thead>
<tr>
<th>( t ) (s)</th>
<th>( v ) (mi/h)</th>
<th>( t ) (s)</th>
<th>( v ) (mi/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>60</td>
<td>65</td>
</tr>
<tr>
<td>10</td>
<td>37</td>
<td>70</td>
<td>55</td>
</tr>
<tr>
<td>20</td>
<td>66</td>
<td>80</td>
<td>64</td>
</tr>
<tr>
<td>30</td>
<td>69</td>
<td>90</td>
<td>44</td>
</tr>
<tr>
<td>40</td>
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<td>100</td>
<td>49</td>
</tr>
<tr>
<td>50</td>
<td>68</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. 1.5 miles  

b. 1.7 miles  

c. 1.8 miles  

d. 1.4 miles  

115 The marginal cost of manufacturing \( x \) yards of a certain fabric is \( C'(x) = 3 - 0.01x + 0.000006x^2 \) (in dollars per yard). Find the increase in cost if the production level is raised from 500 yards to 4500 yards.

a. $95000.00  

b. $96000.00  

c. $94000.00  

d. $97000.00  

116 Evaluate the integral.

\[
\int \left( x^2 + 4 + \frac{1}{x^2 + 1} \right) dx
\]

a. \( x^3 + 4 + \tan^{-1} x + c \)  

b. \( \frac{x^3}{3} + 4x + \tan^{-1} x + c \)  

c. \( \frac{x^3}{3} + 4x + \frac{3}{x^3 + 3} + c \)
Evaluate the integral by making the given substitution:

\[ \int \cos 5x \, dx, \quad u = 5x \]

a. \( \frac{1}{5} \sin 5x \)
b. \( \frac{1}{5} \cos 5x + C \)
c. \( \sin 5x + C \)
d. \( \frac{1}{5} \sin x + C \)
e. \( -\frac{1}{5} \sin 5x + C \)
f. \( \frac{1}{5} \sin 5x + C \)

Evaluate the integral by making the given substitution:

\[ \int x^2 \sqrt{x^3 + 5} \, dx, \quad u = x^3 + 5 \]

a. \( \frac{2}{9} \left( x^3 + 5 \right)^{\frac{3}{2}} \)
b. \( \frac{1}{9} \left( x^3 + 5 \right)^{\frac{1}{2}} + C \)
c. \( \frac{2}{9} \left( x^3 + 5 \right)^{\frac{1}{2}} + C \)
d. \( -\frac{2}{9} \left( x^3 + 5 \right)^{\frac{3}{2}} + C \)
e. \( \frac{2}{9} \left( x^3 + 5 \right)^{\frac{3}{2}} + C \)
Evaluate the integral by making the given substitution:

\[ \int \frac{42}{(1 + 7x)^3} \, dx, \quad u = 1 + 7x \]

a. \( 3 \frac{1}{(1 + 7x)^2} + C \)

b. \( -3 \frac{1}{(1 + 7x)^2} + C \)

c. \( -3 \frac{1}{(1 + 7x)^4} + C \)

d. \( -3 \frac{1}{(1 + 7x)^4} \)

e. \( -6 \frac{1}{(1 + 7x)^2} + C \)

Evaluate the indefinite integral:

\[ \int 4x(x^2 + 7)^4 \, dx \]

a. \( \frac{2}{5} (x^2 + 7)^3 \)

b. \( \frac{2}{5} (x^2 + 7)^5 + C \)

c. \( \frac{4}{5} (x^2 + 7)^5 + C \)

d. \( (x^2 + 7)^5 + C \)

e. \( \frac{2}{5} (x^2 + 7)^4 + C \)

f. \( \frac{2}{5} (x^2 + 7)^3 + C \)
121 Evaluate the indefinite integral:

\[ \int \frac{4 + 6x}{\sqrt{9 + 4x + 3x^2}} \, dx \]

a. \( \sqrt{9 + 4x + 3x^2} + C \)

b. \( 3\sqrt{9 + 4x + 3x^2} + C \)

c. \( -2\sqrt{9 + 4x + 3x^2} \)

d. \( -2\sqrt{9 + 4x + 3x^2} + C \)

e. \( 2\sqrt{9 + 4x + 3x^2} + C \)

122 Evaluate the indefinite integral:

\[ \int t^2 \cos \left(9 - t^3\right) \, dt \]

a. \( \frac{1}{3} \cos \left(9 - t^3\right) + C \)

b. \( -\frac{1}{2} \sin \left(9 - t^3\right) + C \)

c. \( -\sin \left(9 - t^3\right) + C \)

d. \( \frac{1}{3} \sin \left(9 - t^3\right) + C \)

e. \( -\frac{1}{3} \sin \left(9 - t^3\right) + C \)

f. \( \frac{1}{3} \sin \left(9 - t^3\right) \)
Evaluate the indefinite integral:

\[ \int \cos^3 x \sin x \, dx \]

a. \( -\frac{1}{4} \sin^4 x + C \)
b. \( \frac{1}{4} \sin^4 x + C \)
c. \( \frac{1}{4} \cos^4 x + C \)
d. \( -\frac{1}{4} \cos^3 x + C \)
e. \( \frac{1}{4} \sin^3 x + C \)
f. \( -\frac{1}{4} \cos^4 x + C \)

Evaluate the definite integral:

\[ \int_0^1 x^2 (3 + 4x^3)^2 \, dx \]

a. \( \frac{319}{36} \)
b. \( \frac{316}{37} \)
c. \( \frac{316}{39} \)
d. \( \frac{321}{36} \)
e. \( \frac{316}{38} \)
f. \( \frac{316}{36} \)
125 Evaluate the definite integral:
\[ \int_{0}^{\pi/8} \sin 8t \, dt \]

a. 0.25  
b. 0.35  
c. –0.25  
d. 3.25  
e. –1.75  
f. 1.25

126 Find the area of the region that lies under the given curve:
\[ y = \sqrt{5x + 1}, \quad 0 \leq x \leq 1 \]

a. 1.701  
b. 1.936  
c. 1.806  
d. 1.926  
e. 1.829  
f. 1.826

127 If \( f \) is continuous and \( \int_{0}^{30} f(x) \, dx = 10 \), find \( \int_{0}^{6} f(5x) \, dx \).

a. 7  
b. –3  
c. 4  
d. 0  
e. 2  
f. 12
128 Evaluate the indefinite integral:

\[ \int \frac{e^x}{e^x + 8} \, dx \]

a. \(-\frac{1}{2} \ln\left(e^x + 8\right) + C\)
b. \(-\ln\left(e^x + 8\right) + C\)
c. \(\frac{1}{2} \ln\left(e^x + 8\right) + C\)
d. \(\ln\left(e^x + 8\right) + C\)
e. \(\ln\left(e^x - 8\right) + C\)

129 Evaluate the definite integral:

\[ \int_{e^{25}}^{e^{81}} \frac{dx}{x\sqrt{\ln x}} \]

a. 8.1
b. 9
c. 8
d. 7.5
e. 11
f. 6

130 Sketch the region enclosed by the curves \(y = x + 4\), \(y = 36 - x^2\), \(x = -4\), and \(x = 2\). Decide whether to integrate with respect to \(x\) or \(y\). Draw a typical approximating rectangle and label its height and width. Then find the area of the region.

a. 176
b. 43.5
c. 348
d. 174
e. 696
f. 29
Sketch the region enclosed by $y = 7x + 2$ and $y = 7x^2$. Decide whether to integrate with respect to $x$ or $y$. Draw a typical approximating rectangle and label its height and width. Then find the area of the region.

a. 10.98
b. 5.66
c. 0.73
d. 14.64
e. 1.22
f. 3.66

Sketch the region enclosed by $y = 6 + \sqrt{x}$ and $y = \frac{1 + x}{9}$. Decide whether to integrate with respect to $x$ or $y$. Draw a typical approximating rectangle and label its height and width. Then find the area of the region.

a. 1493.503336
b. 2987.006671
c. 746.751668
d. 748.751668
e. 186.687917
f. 373.375834

Sketch the region enclosed by $y = 8x^2$ and $y = x^2 + 4$. Decide whether to integrate with respect to $x$ or $y$. Draw a typical approximating rectangle and label its height and width. Then find the area of the region.

a. 4.031621
b. 16.126484
c. 0.671937
d. 8.063242
e. 1.007905
f. 5.031621

Sketch the region enclosed by $x = 4 - y^2$ and $x = y^2 - 1$. Decide whether to integrate with respect to $x$ or $y$. Draw a typical approximating rectangle and label its height and width. Then find the area of the region.

a. 52.704628
b. 12.540926
c. 21.081851
d. 1.756821
e. 5.270463
f. 10.540926
135 Use calculus to find the area of the triangle with the given vertices.

\((0, 0), (7, 1), (-1, 5)\)

a. \(S = 19\)
b. \(S = 18\)
c. \(S = 19.5\)
d. \(S = 17.5\)

136 Use the Midpoint Rule with \(n = 4\) to approximate the area of the region bounded by the given curves.

\[y = \sqrt[3]{1 + x^3}, \ y = 1 - 6x, \ x = 2\]

a. \(S = 11.22\)
b. \(S = 14.22\)
c. \(S = 13.22\)
d. \(S = 10.22\)

137 Use a graph to find approximate \(x\) coordinates of the points of interception of the curves. Then find (approximately) the area of the region bounded by the curves.

\[y = 2\cos x, \ y = x^4\]

a. \(A \approx 3.83\)
b. \(A \approx 1.48\)
c. \(A \approx 3.42\)
d. \(A \approx 2.97\)
Racing cars driven by Chris and Kelly are side by side at the start of a race. The table shows the velocities of each car (in miles per hour) during the first ten seconds of the race. Use the Midpoint Rule to estimate how much farther Kelly travels than Chris does during the ten seconds.

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a. 115 \( \frac{1}{3} \) ft
b. 114 \( \frac{2}{3} \) ft
c. 118 \( \frac{1}{3} \) ft
d. 117 \( \frac{1}{3} \) ft

Find the area of the region bounded by the parabola \( y = x^2 \), the tangent line to this parabola at (4, 16), and the x-axis.

a. 23.333333
b. 27.333333
c. 21.333333
d. 25.333333
e. 24.333333

Find the number \( b \) such that the line \( y = b \) divides the region bounded by the curves \( y = 10x^2 \) and \( y = 8 \) into two regions with equal area.

a. \( \frac{80}{2^{3/2}} \)
b. \( \frac{80}{2^{3/3}} \)
c. \( \frac{8}{2^{3/2}} \)
d. \( \frac{8}{2^{-3/2}} \)
e. \( \frac{80}{2^{-3/2}} \)
f. \( \frac{8}{2^{2/3}} \)
Find the positive value of $c$ such that the area of the region bounded by the parabolas $y = x^2 - c^2$ and $y = c^2 - x^2$ is 1944.

a. 7
b. 9
c. 10
d. 16
e. 8

Find the volume of the solid obtained by rotating about the $x$– axis the region under the curve $y = \frac{1}{x}$ from $x = 3$ to $x = 8$.

a. $\frac{11}{24}\pi$
b. $\frac{5}{24}\pi$
c. $11\pi$

d. $5\pi$

Find the volume of the solid obtained by rotating the region in the first quadrant bounded by $y = x^2$ and $y = 5$ about the $y$ – axis.

a. $\frac{25}{2}\pi$
b. $\frac{25}{2}$
c. $\frac{5}{2}\pi$

d. $\frac{5}{2}$

Find the volume of the solid obtained by rotating the region bounded by $y = x^4$ and $x = y^4$ about the $x$ – axis.

a. $\frac{5}{9}\pi$
b. $\frac{16}{27}\pi$
c. $\frac{10}{3}\pi$
Find the volume of the solid obtained by rotating the region bounded by \( x = y^2 \) and \( x = 3y \) about the \( y \)-axis.

a. \( \frac{-243}{3} \pi \)

b. \( \frac{243}{15} \pi \)

c. \( \frac{1458}{15} \pi \)

Find the volume of the solid obtained by rotating the region bounded by \( y = \frac{3}{\sqrt{x}} \) and \( y = x \) about the line \( y = 1 \).

a. \( \frac{4}{15} \pi \)

b. \( \frac{4}{15} \)

c. \( \frac{6}{15} \pi \)

d. \( \frac{6}{15} \)

Find the volume of the solid obtained by rotating the region bounded by \( y = x^4 \) and \( x = y^4 \) about the line \( x = -1 \).

a. \( \frac{498}{54} \pi \)

b. \( \frac{474}{30} \pi \)

c. \( \frac{474}{270} \pi \)

Find the volume of a right circular cone with height \( h = 6 \) and base radius \( r = 2 \).

a. \( 8 \pi \)

b. \( 8 \)

c. \( 24 \pi \)
Find the volume of a cap of a sphere with radius $r = 3$ and height $h = 0.45$.

a. $1.2825\pi$  
b. $0.6075\pi$  
c. $0.577125\pi$

Find the volume of a pyramid with height $h = 18$ and rectangular base with dimensions 5 and 10.

a. 300  
b. 60  
c. 150

Find the volume of a pyramid with height 9 and base an equilateral triangle with side $a = 3$.

a. 11.69  
b. 23.38  
c. 46.77  
d. 3.9
152 Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the y–axis.

\[ y = \frac{1}{x}, \quad y = 0, \quad x = 1, \quad x = 4 \]

a. \( V = 7\pi \)
b. \( V = 6\pi \)
c. \( V = 2\pi \)
d. \( V = 3\pi \)

153 Use the method of cylindrical shells to find the volume of solid obtained by rotating the region bounded by the given curves about the x–axis.

\[ x = 4 + y^2, \quad x = 0, \quad y = 1, \quad y = 3 \]

a. \( V = 144\pi \)
b. \( V = 70\pi \)
c. \( V = 77\pi \)
d. \( V = 72\pi \)

154 Use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by the given curves about the x–axis.

\[ y^2 - 12y + x = 0, \quad x = 0 \]

a. \( V = 3506\pi \)
b. \( V = 3436\pi \)
c. \( V = 3456\pi \)
d. \( V = 6912\pi \)

155 Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the specified axis.

\[ y = x^2, \quad y = 0, \quad x = 1, \quad x = 8; \quad \text{about } x = 1 \]

a. \( V = 20482\pi \)
b. \( V = 10246\pi \)
c. \( V = 10239\pi \)
d. \( V = \frac{10241}{6}\pi \)
Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the specified axis.

\[ y = \sqrt{x - 1}, \quad y = 0, \quad x = 17; \quad \text{about } y = 9 \]

a. \( V = 640 \pi \)
b. \( V = 645 \pi \)
c. \( V = 638 \pi \)
d. \( V = 1280 \pi \)

Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis.

\[ y = \sin x, \quad y = 0, \quad x = 4 \pi, \quad x = 6 \pi; \quad \text{about the } y - \text{axis.} \]

a. \( V = \int_{0}^{4 \pi} 2 \pi x \sin(x) \, dx \)
b. \( V = \int_{4 \pi}^{6 \pi} 2 \pi x \sin(x) \, dx \)
c. \( V = \int_{4 \pi}^{6 \pi} 2 \pi x \sin(x) \, dx \)
d. \( V = \int_{4 \pi}^{6 \pi} 2 \pi \sin(x) \, dx \)

Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis.

\[ x = \sqrt{\sin y}, \quad 0 \leq y \leq \pi, \quad x = 0; \quad \text{about } y = 4. \]

a. \( V = \int_{0}^{\pi} 2 (4 - y) \sqrt{\sin y} \, dy \)
b. \( V = \int_{0}^{\pi} (4 - y) \sqrt{\sin y} \, dy \)
c. \( V = \int_{0}^{\pi} 2 (4 - y) \sqrt{\sin y} \, dy \)
d. \( V = \int_{0}^{\pi} (4 - y) \sqrt{\sin y} \, dy \)
159 Use the Midpoint Rule with \( n = 4 \) to estimate the volume obtained by rotating about \( y \)-axis the region under the curve

\[
y = \tan x, \quad 0 \leq x \leq \frac{\pi}{4}\.
\]

a. \( V = 1.560 \)
b. \( V = 1.824 \)
c. \( V = 1.142 \)
d. \( V = 0.548 \)

160 Sketch a graph to estimate the \( x \)-coordinates of the points of intersection of the given curves. Then use this information to estimate the volume of the solid obtained by rotating about the \( y \)-axis the region enclosed by these curves. Please round the answers to the nearest hundredth.

\[
y = 0, \quad y = -x^4 + 5x^3 - x^2 + 5x
\]

a. \( V = 1145.83 \pi \)
b. \( V = 1151.06 \pi \)
c. \( V = 1166.39 \pi \)
d. \( V = 1143.31 \pi \)

161 The region bounded by the given curves is rotated about the specified axis. Find the volume of the resulting solid by any method.

\[
y = x^2 - 2x - 3; \text{ about the } x \text{-axis.}
\]

a. \( V = 54.47 \pi \)
b. \( V = 39.58 \pi \)
c. \( V = 34.13 \pi \)
d. \( V = 31.63 \pi \)

162 The region bounded by the given curves is rotated about the specified axis. Find the volume of the resulting solid by any method.

\[
y = 5, \quad y = x^2 - 6x + 13; \text{ about } x = -1
\]

a. \( V = 8.17 \pi \)
b. \( V = 10.67 \pi \)
c. \( V = 16.12 \pi \)
d. \( V = 31.01 \pi \)
163 Use cylindrical shells to find the volume of the solid.
A sphere of radius \( k \).

a. \( V = \frac{4}{3} \pi k^3 \)
b. \( V = \frac{5}{3} \pi k^3 \)
c. \( V = \frac{1}{3} \pi k^3 \)
d. \( V = \frac{2}{3} \pi k^3 \)

164 Use cylindrical shells to find the volume of the solid.
A right circular cone with height \( f \) and base radius \( d \).

a. \( V = \frac{\pi df^2}{3} \)
b. \( V = \frac{\pi fd^2}{2} \)
c. \( V = \frac{\pi fd^3}{3} \)
d. \( V = \frac{\pi fd^2}{3} \)

165 Suppose you make napkin rings by drilling holes with different diameters through two wooden balls (which also have different diameters). You discover that both napkin rings have the same height \( h \) as shown in the figure.
Use cylindrical shells to compute the volume of a napkin ring created by drilling a hole with radius \( k \) through the center of a sphere of radius \( K \) and express the answer in terms of \( h \).

a. \( V = \frac{1}{4} \pi h^3 \)
b. \( V = \frac{1}{6} \pi h^2 \)
c. \( V = \frac{1}{6} \pi h^3 \)
d. \( V = \frac{1}{3} \pi h^2 \)
Find the work done in pushing a car a distance of 12 m while exerting a constant force of 900 N.

a. \( W = 9300 \text{ J} \)  
b. \( W = 10800 \text{ J} \)  
c. \( W = 10500 \text{ J} \)  
d. \( W = 10300 \text{ J} \)

A particle is moved along the \( x \)-axis by a force that measures \( \frac{12}{(1 + x)^2} \) pounds at a point \( x \) feet from the origin. Find the work done in moving the particle from the origin to a distance of 11 ft.

a. 22 lb – ft  
b. 11 lb – ft  
c. 13 lb – ft  
d. –11 lb – ft  
e. 0 lb – ft

If 57 J of work are needed to stretch a spring from 10 cm to 13 cm and another 135 J are needed to stretch it from 13 cm to 18 cm, what is the natural length of the spring?

a. 2  
b. 1  
c. 0  
d. 3

A heavy rope, 20 ft long, weighs 0.5 lb/ft and hangs over the edge of a building 120 ft high. How much work is done in pulling the rope to the top of the building?

a. –51 ft – lb  
b. 101 ft – lb  
c. 200 ft – lb  
d. 100 ft – lb  
e. 50 ft – lb
170 A bucket that weighs 3 lb and a rope of negligible weight are used to draw water from a well that is 90 ft deep. The bucket starts with 30 lb of water and is pulled up at a rate of 5 ft/s, but water leaks out of a hole in the bucket at a rate of 0.5 lb/s. Find the work done in pulling the bucket to the top of the well.

a. 2566 ft-lb
b. 2565 ft-lb
c. 2675 ft-lb
d. 2515 ft-lb

171 An aquarium 4 m long, 9 m wide, and 1 m deep is full of water. Find the work needed to pump half of the water out of the aquarium. (Use the facts that the density of water is \(1000 \text{ kg/m}^3\) and \(g \approx 9.8 \text{ } \text{m/s}^2\).)

a. 43950 J
b. 44111 J
c. 44100 J
d. 44110 J
e. 43100 J

172 A tank is full of water. Find the work required to pump the water out of the outlet. Round the answer to the nearest thousand.

\[ h = 2 \text{ m} , \ r = 2 \text{ m} , \ d = 7 \text{ m} \]

a. \( W = 3895000 \text{ J} \)
b. \( W = 3168000 \text{ J} \)
c. \( W = 3446000 \text{ J} \)
d. \( W = 3569000 \text{ J} \)
The tank shown is full of water. Given that water weighs 62.5 lb/ft and \( R = 3 \), find the work required to pump the water out of the tank.

![Hemisphere diagram]

\[ W = \int_{V_1}^{V_2} PDV \]

a. 3996 ft·lb
b. 2975 ft·lb
c. 3976 ft·lb
d. 2976 ft·lb
e. 3866 ft·lb

When gas expands in a cylinder with radius \( r \), the pressure at any given time is a function of the volume: \( P = P(V) \). The force exerted by the gas on the piston (see the figure) is the product of the pressure and the area: \( F = \pi r^2 P \). Find the work done by the gas when the volume expands from volume \( V_1 \) to volume \( V_2 \).

![Cylinder diagram]

a. \( W = \int_{V_1}^{V_2} VdP \)
b. \( W = \int_{V_1}^{V_2} VdP \)
c. \( W = \int_{V_1}^{V_2} PdV \)
d. \( W = \int_{V_1}^{V_2} VdP \)
e. \( W = \int_{V_1}^{V_2} PdV \)
In a steam engine the pressure and volume of steam satisfy the equation $PV^{1.4} = k$, where $k$ is a constant. (This is true for adiabatic expansion, that is, expansion in which there is no heat transfer between the cylinder and its surroundings.) Calculate the work done by the engine during a cycle when the steam starts at a pressure of 150 lb/in$^2$ and a volume of 100 in$^3$ and expands to a volume of 800 in$^3$.

Use the fact that the work done by the gas when the volume expands from volume $V_1$ to volume $V_2$ is

$$W = \int_{V_1}^{V_2} P \, dV$$

a. 1875 ft \cdot lb
b. 766 ft \cdot lb
c. 1765 ft \cdot lb
d. 1715 ft \cdot lb
e. 1766 ft \cdot lb

176 Newton's Law of Gravitation states that two bodies with masses $m_1$ and $m_2$ attract each other with a force

$$F = G \frac{m_1 m_2}{r^2}$$

where $r$ is the distance between the bodies and $G$ is the gravitation constant. If one of the bodies is fixed, find the work needed to move the other from $r = a$ to $r = d$.

a. $W = G m_1 m_2 \left( \frac{1}{d^2} - \frac{1}{a^2} \right)$
b. $W = G m_1 m_2 \left( \frac{1}{d} - \frac{1}{a} \right)$
c. $W = G m_1 m_2 \left( \frac{1}{a} - \frac{1}{d} \right)$
d. $W = G m_1 m_2 \left( \frac{1}{a^2} - \frac{1}{d^2} \right)$

d. $W = G m_1 m_2 \left( \frac{1}{a^2} - \frac{1}{d^2} \right)$

177 Find the average value of the function $y(\nu) = 4 \cos \nu$ on the interval $[0, \frac{\pi}{2}]$.

a. $\frac{4}{\pi}$
b. $\frac{16}{\pi}$
c. $\frac{8}{\pi}$
Find the average value of the function \( z(t) = 2 \sqrt{t} \) on the interval \([1, 25]\).

a. \( \frac{496}{72} \)
b. \( \frac{496}{216} \)
c. \( \frac{1488}{24} \)
d. \( \frac{496}{24} \)

Find a number \( b \) in \([0, 8]\) at which the value of the function \( f(t) = 4 - t^2 \) is equal to the average value of the function on the interval \([0, 8]\).

a. \( b = -\frac{8 \sqrt{3}}{3} \)
b. \( b = \frac{8 \sqrt{2}}{3} \)
c. \( b = \frac{8 \sqrt{3}}{3} \)
d. \( b = \frac{16 \sqrt{3}}{3} \)

Find the average value of the function \( y(\nu) = 5 \nu \sin(\nu^2) \) on the interval \([0, \sqrt{\pi}]\).

a. \( \frac{10}{\pi} \)
b. \( \frac{5}{\pi} \)
c. \( \frac{10}{\sqrt{\pi}} \)
d. \( \frac{5}{\sqrt{\pi}} \)

Find the number(s) \( b \) such that the average value of the function \( z(\nu) = 37 - 24 \nu + 3 \nu^2 \) on the interval \([0, b]\) is equal to 2.

a. \( -5 \)
b. \( 7 \)
c. \( 5 \)
d. \( 2 \)
In a certain city the temperature $v$ hours after 6 A.M. was modeled by the function

$$U(v) = 42 + 27 \sin \frac{\pi v}{15}$$

Find the average temperature during the period from 6 A.M. to 6 P.M.

a. 61.434  
b. 3.434  
c. 38.858  
d. 36.13

The velocity $v$ of blood that flows in a blood vessel with radius $R$ and length $l$ at a distance $r$ from the central axis is

$$v(r) = \frac{P}{4q l} \left( R^2 - r^2 \right)$$

where $P$ is the pressure difference between the ends of the vessel and $q$ is the viscosity of the blood.

Suppose that Vessel 1 has length 2.1 cm and outer radius 1.9 mm, and Vessel 2 has length 4.4 cm and outer radius 7.8 mm. Find the average velocities (with respect to $r$) over the interval $0 \leq r \leq R$ for each vessel. Which vessel has the higher average velocity?

a. Vessel 1  
b. Vessel 2
| 1. a | 22. b | 23. c | 24. a | 25. c | 26. b,c,d | 27. a,d | 28. e | 29. d | 30. c,e | 31. d | 32. a | 33. b | 34. c | 35. b | 36. a | 37. c | 38. b | 39. c | 40. d | 41. d | 42. a | 43. a | 44. a | 45. c | 46. c | 47. c | 48. b | 49. d | 50. d | 51. b | 52. f | 53. e | 54. e | 55. f | 56. e | 57. e | 58. c | 59. a | 60. d | 61. d | 62. e | 63. f | 64. d | 65. d | 66. f | 67. a | 68. b | 69. c | 70. b | 71. b | 72. b | 73. b | 74. b | 75. a | 76. b | 77. b | 78. b | 79. a | 80. a | 81. b | 82. a | 83. c | 84. c | 85. b | 86. a | 87. a | 88. a | 89. d | 90. c | 91. c | 92. a | 93. e | 94. d | 95. b | 96. e | 97. a | 98. b | 99. a | 100. b | 101. c | 102. d | 103. d | 104. e | 105. c | 106. c | 107. d | 108. b | 109. a | 110. e | 111. b | 112. d | 113. c | 114. a | 115. c | 116. b | 117. f | 118. e | 119. b | 120. b | 121. e | 122. e | 123. f | 124. f | 125. a | 126. f | 127. e | 128. d | 129. c | 130. d | 131. f | 132. c | 133. a | 134. f | 135. b | 136. c | 137. d | 138. c | 139. c | 140. c | 141. b | 142. b | 143. a | 144. a | 145. c | 146. a | 147. c | 148. a | 149. c | 150. a | 151. a | 152. b | 153. d | 154. c | 155. d | 156. a | 157. c | 158. c | 159. c | 160. d | 161. c | 162. b | 163. a | 164. d | 165. c | 166. b | 167. b | 168. a | 169. d | 170. b | 171. c | 172. c | 173. c | 174. e | 175. c | 176. c | 177. c | 178. a | 179. c | 180. d | 181. b,c | 182. a | 183. b |
1. Find two positive numbers whose product is 64 and whose sum is a minimum.
   a. 2, 32
   b. 4, 16
   c. 8, 8

2. Consider the following problem: A farmer with 780 ft of fencing wants to enclose a rectangular area and then divide it into four pens with fencing parallel to one side of the rectangle. What is the largest possible total area of the four pens?
   a. 15310 ft²
   b. 15233 ft²
   c. 15230 ft²
   d. 15199 ft²
   e. 15209 ft²
   f. 15210 ft²

3. If 2500 cm² of material is available to make a box with a square base and an open top, find the largest possible volume of the box.
   a. 12028 cm³
   b. 12027 cm³
   c. 12017 cm³
   d. 12048 cm³
   e. 12128 cm³
   f. 12051 cm³

4. A rectangular storage container with an open top is to have a volume of 10 m³. The length of its base is twice the width. Material for the base costs $13 per square meter. Material for the sides costs $6 per square meter. Find the cost of materials for the cheapest such container.
   a. $176.19
   b. $176.99
   c. $182.69
   d. $178.51
   e. $178.49
   f. $177.49
Find the point on the line $y = 8x + 3$ that is closest to the origin.

a. $\left(\frac{-24}{65}, \frac{5}{65}\right)$

b. $\left(\frac{-24}{64}, \frac{4}{64}\right)$

c. $\left(\frac{-26}{65}, \frac{4}{65}\right)$

d. $\left(\frac{-24}{64}, \frac{3}{64}\right)$

e. $\left(\frac{-23}{65}, \frac{3}{65}\right)$

f. $\left(\frac{-24}{65}, \frac{3}{65}\right)$

Find the dimensions of the rectangle of largest area that can be inscribed in an equilateral triangle of side $L = 9$ cm if one side of the rectangle lies on the base of the triangle. Round the result to the nearest tenth.

a. 4.5 cm, 4 cm

b. 5.5 cm, 4.4 cm

c. 9.5 cm, 3.9 cm

d. 4 cm, 3.91 cm

e. 4.5 cm, 3.9 cm

f. 7.5 cm, 2.9 cm

A right circular cylinder is inscribed in a sphere of radius $r = 5$ cm. Find the largest possible surface area of such a cylinder. Round the result to the nearest hundredth.

a. 254.66 cm$^2$

b. 253.05 cm$^2$

c. 254.06 cm$^2$

d. 254.18 cm$^2$

e. 254.16 cm$^2$

f. 259.16 cm$^2$
8 A Norman window has the shape of a rectangle surmounted by a semicircle. (Thus the diameter of the semicircle is equal to the width of the rectangle.) If the perimeter of the window is 37 ft, find the dimensions of the window so that the greatest possible amount of light is admitted. Round the result to the nearest hundredth.

a. base = 11.36 ft, height = 5.15 ft
b. base = 10.46 ft, height = 5.38 ft
c. base = 10.34 ft, height = 6.18 ft
d. base = 10.36 ft, height = 5.18 ft
e. base = 10.47 ft, height = 5.18 ft
f. base = 10.36 ft, height = 4.68 ft

9 A piece of wire 19 m long is cut into two pieces. One piece is bent into a square and the other is bent into an equilateral triangle. How should the wire be cut for the square so that the total area enclosed is a minimum? Round the result to the nearest hundredth.

a. 0 m
b. 7.16 m
c. 8.31 m
d. 19 m
e. 8.26 m
f. 9.26 m

10 A fence 8 ft tall runs parallel to a tall building at a distance of 2 ft from the building. What is the length of the shortest ladder that will reach from the ground over the fence to the wall of the building? Round the result to the nearest hundredth.

a. 13.41 ft
b. 13.21 ft
c. 12.11 ft
d. 13.20 ft
e. 15.24 ft
f. 14.21 ft
A conical drinking cup is made from a circular piece of paper of radius $R = 5$ cm by cutting out a sector and joining the edges $CA$ and $CB$. Find the maximum capacity of such a cup. Round the result to the nearest hundredth.

a. $50.38 \text{ cm}^3$
b. $50.4 \text{ cm}^3$
c. $49.37 \text{ cm}^3$
d. $50.43 \text{ cm}^3$
e. $50.48 \text{ cm}^3$
f. $49.38 \text{ cm}^3$
A woman at a point $A$ on the shore of a circular lake with radius 4 mi wants to arrive at the point $C$ diametrically opposite on the other side of the lake in the shortest possible time. She can walk at the rate of 5 mi/h and row a boat at 1 mi/h. How should she proceed? (Find $\theta$). Round the result, if necessary, to the nearest hundredth.

a. She should row from point $A$ to point $C$ radians
b. 0.53 radians
c. 0.41 radians
d. She should walk around the lake from point $A$ to point $C$.
e. 0.2 radians
f. 0.25 radians

Find an equation of the line through the point $(7, 28)$ that cuts off the least area from the first quadrant.

a. $y = -4x + 57$
b. $y = 4x + 56$
c. $y = -5x + 56$
d. $y = -4x + 56$
e. $y = -5x + 57$
14 Consider the figure below, where \( a = 7 \), \( b = 1 \) and \( l = 7 \). How far from the point \( A \) should the point \( P \) be chosen on the line segment \( AB \) so as to maximize the angle \( \theta \)? Round the result to the nearest hundredth.

\[ \text{Options:} \]
- a. 4.8
- b. 4.1
- c. 4.14
- d. 5.4
- e. 5.11
- f. 4.27

15 A painting in an art gallery has height \( h = 67 \) cm and is hung so that its lower edge is a distance \( d = 18 \) cm above the eye of an observer (as seen in the figure below). How far from the wall should the observer stand to get the best view? (In other words, where should the observer stand so as to maximize the angle \( \theta \) subtended at his eye by the painting?) Round the result to the nearest hundredth.

\[ \text{Options:} \]
- a. 37.76 cm
- b. 41.64 cm
- c. 42.47 cm
- d. 38 cm
- e. 39.21 cm
- f. 39.12 cm
A steel pipe is being carried down a hallway 15 ft wide. At the end of the hall there is a right - angled turn into a narrower hallway 12 ft wide. What is the length of the longest pipe that can be carried horizontally around the corner? Round the result to the nearest hundredth.

\[ \theta \]

- a. 37.07 ft
- b. 38.13 ft
- c. 38.1 ft
- d. 36.9 ft
- e. 38.15 ft
- f. 38.8 ft

Find the maximum area of a rectangle that can be circumscribed about a given rectangle with length \( L = 5 \) and width \( W = 7 \).

\[ \theta \]

- a. 75
- b. 71
- c. 69
- d. 82
- e. 72.5
- f. 72
18 The graph of a function \( f(x) \) is given below. Suppose that Newton's method is used to approximate the roots \( r \) and \( s \) of the equation \( f(x) = 0 \). The blue (1) and red (2) lines are the tangent lines corresponding to initial approximations for finding these roots.

In the approximation of the value of \( r \), what is \( x_2 \)?

\[ f(x) \]

\[ \begin{array}{c}
\text{f} \\
20 \\
10 \\
\text{f}
\end{array} \]

\[ x \]

\[ \begin{array}{c}
4 \\
9 \\
12 \\
16 \\
20
\end{array} \]

- a. \( x_2 \approx 6 \)
- b. \( x_2 \approx 11.8 \)
- c. \( x_2 \approx 11 \)
- d. \( x_2 \approx 5.2 \)

19 Suppose the line \( y = 16x - 5 \) is tangent to the curve \( y = f(x) \) when \( x = -3 \). If Newton's method is used to locate a root of the equation \( f(x) = 0 \) and the initial approximation is \( x_1 = -3 \), find the second approximation \( x_2 \).

- a. \( x_2 = \frac{5}{16} \)
- b. \( x_2 = \frac{7}{16} \)
- c. \( x_2 = -\frac{5}{16} \)
20 The graph of a function is given. For which of the following initial approximations does Newton's method fail?

a. $x_1 = 7$
b. $x_1 = 1$
c. $x_1 = 2$
d. $x_1 = 8$

21 Use Newton's method with the specified initial approximation $x_1$ to find $x_4$, the fourth approximation to the root of the given equation. (Give your answer to five decimal places.)

$$x^3 + x^2 - 9 = 0, \ x_1 = 10$$

a. $x_4 = 5.89449$
b. $x_4 = 1.41107$
c. $x_4 = 4.35557$
d. $x_4 = 2.94449$

22 Use Newton's method with the specified initial approximation $x_1$ to find $x_3$, the third approximation to the root of the given equation. (Give your answer to four decimal places.)

$$x^4 - 12 = 0, \ x_1 = 5$$

a. $x_3 = 7.5763$
b. $x_3 = 2.8863$
c. $x_3 = 4.4496$
d. $x_3 = 3.7740$
23 Use Newton's method to approximate the given number, correct to eight decimal places:

\[ \frac{10}{\sqrt{17}} \]

a. 1.46963602  

b. 1.46963600  

c. 1.46963599  

d. 1.46963601

24 Use Newton's method to approximate the root of

\[ x^4 + x - 10 = 0 \]

in the interval [1, 2], correct to six decimal places.  
Use \( x_1 = 1.5 \) as the initial approximation.

a. \( x = 1.697473 \)  

b. \( x = 1.697472 \)  

c. \( x = 1.697469 \)  

d. \( x = 1.697470 \)

25 Newton's method is used to approximate the least positive root of the equation:

\[ 9 \sin x = x \]

Using an initial approximation \( x_1 = 1.96 \), find the fifth approximation \( x_5 \).

a. \( x_5 = 2.822589 \)  

b. \( x_5 = 2.822587 \)  

c. \( x_5 = 2.822588 \)  

d. \( x_5 = 2.82259 \)
26 Use Newton's method to find all the roots of the equation, correct to six decimal places.

\[ 4x^5 - 8x^4 - 26x^3 - 16x^2 - 30x - 8 = 0 \]

a. \( x = -1.707107 \)

b. \( x = -0.292893 \)

c. \( x = -3.414214 \)

d. \( x = -0.585786 \)

e. \( x = 4 \)

27 Use Newton's method to find all the roots of the equation, correct to four decimal places.

\[ x^4 - 4x^2 - 140 + \frac{12}{x^2 + 7} = \frac{x^2 + 19}{x^2 + 7} - 1 \]

a. \( x = -3.7417 \)

b. \( x = 3.7418 \)

c. \( x = -3.7407 \)

d. \( x = 3.7417 \)

e. \( x = 3.7407 \)

28 The following algorithm used by the ancient Babylonians to compute \( \sqrt{a} \):

\[ x_{n+1} = \frac{1}{2} \left( x_n + \frac{a}{x_n} \right) \]

(You can derive it by applying Newton's method to the equation \( x^2 - a = 0 \).)

Use this algorithm to compute \( \sqrt{145} \), correct to six decimal places.

a. \( 12.041593 \)

b. \( 12.041594 \)

c. \( 12.041596 \)

d. \( 12.041592 \)

e. \( 12.041595 \)
29 The following algorithm enables a computer to find reciprocals without actually dividing:

\[ x_{n+1} = 2x_n - ax_n^2 \]

(You can derive it by apply Newton's method to the equation \( \frac{1}{x} - a = 0 \).

Use this algorithm to compute \( \frac{1}{1.5991} \), correct to seven decimal places.

a. 0.6253521  
b. 0.6253518  
c. 0.6253514  
d. 0.6253516

30 The equation \( 6x^3 - 18x + 8 = 0 \) is given.

For some initial approximations, Newton's method doesn't work for finding the root of this equation.

Which of the following are such values of \( x_1 \)?

a. \( x_1 = -8 \)  
b. \( x_1 = 9 \)  
c. \( x_1 = -1 \)  
d. \( x_1 = -3 \)  
e. \( x_1 = 1 \)

31 Find the initial approximation \( x_1 \) for which Newton's method succeeds when applied to the equation \( 8\sqrt[3]{x} = 0 \).

a. \( x_1 = -10 \)  
b. \( x_1 = 0 \)  
c. \( x_1 = -8 \)  
d. \( x_1 = 8 \)

32 A grain silo consists of a cylindrical main section, with height 33 ft, and a hemispherical roof. In order to achieve a total volume of 15000 \( ft^3 \) (including the part inside the roof section), what would the radius of the silo have to be? Find the result and round to four decimal places.

a. \( r = 15.1745 \) ft  
b. \( r = 11.6500 \) ft  
c. \( r = 10.8901 \) ft  
d. \( r = 10.8852 \) ft
33 Find the most general antiderivative of the function:

\[ f(x) = 12x^2 - 12x + 2 \]

a. \[ F(x) = 12x^3 - 12x^2 + 2x + C \]
b. \[ F(x) = 20x^5 - 24x^4 + 2x + C \]
c. \[ F(x) = 4x^3 - 6x^2 + 2x + C \]

34 Find the most general antiderivative of the function:

\[ f(x) = \frac{1}{8} - \frac{1}{5} \]

a. \[ F(x) = \frac{7}{8}x^5 - \frac{4}{5} + C \]
b. \[ F(x) = \frac{9}{8}x^6 - \frac{6}{5} + C \]
c. \[ F(x) = \frac{9}{8}x^6 - \frac{6}{5} + C \]

35 Find the most general antiderivative of the function:

\[ f(x) = \frac{5}{x^6}, \quad x \neq 0 \]

a. \[ F(x) = \frac{1}{5}x^5 + C \]
b. \[ F(x) = -\frac{1}{5}x^5 + C \]
c. \[ F(x) = -\frac{1}{7}x^7 + C \]
36 Find the most general antiderivative of the function:

\[ f(x) = 4\cos x - 4\sin x \]

a. \( F(x) = 4 \sin(x) - 4 \cos(x) + C \)
b. \( F(x) = -4 \sin(x) + 4 \cos(x) + C \)
c. \( F(x) = 4 \sin(x) + 4 \cos(x) + C \)

37 Find \( f \):

\[ f''(x) = 18x + 24x^2 \]

a. \( f(x) = 6x^3 + 8x^4 + Cx + D \)
b. \( f(x) = 9x^3 + 4x^4 + Cx + D \)
c. \( f(x) = 3x^3 + 2x^4 + Cx + D \)

38 Find \( f \):

\[ f''(x) = 4 \cos(2x) \]

a. \( f(x) = y = 4\cos(x) + Cx + D \)
b. \( f(x) = y = -\cos(2x) + Cx^2 + D \)
c. \( f(x) = -\cos(2x) + Cx + D \)

39 Find \( f \):

\[ f'(x) = 5 \cos(x) + 7 \sin(x) \]

\( f(0) = 9 \)

a. \( f(x) = 5\sin(x) - 7\cos(x) + 9 \)
b. \( f(x) = 5\sin(x) + 7\cos(x) + 16 \)
c. \( f(x) = 5\sin(x) - 7\cos(x) + 16 \)
40 Find $f$:

$$f''(x) = 60x$$

$f(4) = 658$

$f'(4) = 484$

- a. $f(x) = 11x^3 + 4x + 3$
- b. $f(x) = 11x^3 + 5x + 2$
- c. $f(x) = 10x^3 + 5x + 3$
- d. $f(x) = 10x^3 + 4x + 2$

41 Given that the graph of $f$ passes through the point $(2, 9)$ and that the slope of its tangent line at $(x, f(x))$ is $6x - 2$, find $f(3)$.

- a. 21
- b. 44
- c. 22
- d. 23

42

The graph of a function $g(x)$ is shown. Which graph is a possible graph for antiderivative of $g(x)$?

- a. 1
- b. 2
- c. 3

43 Evaluate $f(x) = \sin\left(x^2\right)$, and tell whether its antiderivative $F$ increasing or decreasing at the point $x = -4$ radians.

- a. 0.757, decreasing
- b. -0.288, decreasing
- c. -0.288, increasing
- d. 0.757, increasing
44 A particle moves along a straight line with velocity function \( v(t) = 5 \sin(t) - 7 \cos(t) \) and its initial displacement is \( s(0) = 4 \). Find its position function.

a. \( s(t) = 9 + 5 \cos(t) - 7 \sin(t) \)

b. \( s(t) = 9 - 5 \cos(t) - 7 \sin(t) \)

c. \( s(t) = 5 - 5 \cos(t) + 7 \sin(t) \)

d. \( s(t) = 4 - 5 \cos(t) - 7 \sin(t) \)

45 A stone is dropped from the upper observation deck (the Space Deck) of a tower, 240m above the ground. Find the distance of the stone above ground level at time \( t \).

a. \( s(t) = 240 - 9.8t^2 \)

b. \( s(t) = 240 + 4.9t^2 \)

c. \( s(t) = 240 - 4.9t^2 \)

d. \( s(t) = 240 + 9.8t^2 \)

46 A stone was dropped off a cliff and hit the ground with a speed of 320 ft/s. What is the height of the cliff?

for g.

a. 102400 ft

b. 1600 ft

c. 3200 ft

d. 51200 ft

47 A company estimates that the marginal cost (in dollars per item) of producing items is \( 3.02 - 0.016x \). If the cost of producing one item is $587 find the cost of producing 100 items.

a. $805.99

b. $1389.98

c. $1611.98

d. $222.00

48 The linear density of a rod of length 1 m is given by \( \frac{1}{\sqrt{x}} \) in grams per centimeter, when it is measured in centimeters from one end of the rod. Find the mass of the rod.

a. 1 g

b. 20 g

c. 2 g

d. 10 g
49 What constant acceleration is required to increase the speed of a car from 15 ft/s to 20 ft/s in 10 s?

a. \( \frac{350}{s} \) ft/s^2

b. \( \frac{1}{s} \) ft/s^2

c. \( \frac{0.5}{s} \) ft/s^2

d. \( \frac{3.5}{s} \) ft/s^2

50 A car braked with a constant deceleration of 25 ft/s^2, producing skid marks measuring 95 ft before coming to a stop. How fast was the car traveling when the brakes were first applied?

a. 34.46 ft/s

b. 68.92 ft/s

c. 48.73 ft/s

d. 137.84 ft/s

51 By reading values from the given graph of \( f \), use five rectangles to find a lower estimate for the area from \( x = 0 \) to \( x = 10 \) under the given graph of \( f \). Round your answer to the nearest tenth.

\[ \text{graph} \]

a. 31.2

b. 32.1

c. 29.6

d. 28.8

e. 32.9
Estimate to the hundredth the area from 1 to 5 under the graph of $f(x) = \frac{3}{x}$ using four approximating rectangles and right endpoints.

a. 3.77  
b. 4.95  
c. 2.85  
d. 5.83  
e. 3.85  
f. 2.47

Estimate the area from 0 to 5 under the graph of $f(x) = 100 - x^2$ using five approximating rectangles and right endpoints.

a. 451  
b. 447  
c. 465  
d. 435  
e. 440  
f. 445

Approximate the area under the curve $y = \sin x$ from 0 to $\pi/2$ using ten approximating rectangles of equal widths and right endpoints.

a. 2.82  
b. 1.38  
c. 1.08  
d. 1.92  
e. 2.86  
f. 2.20

Approximate the area under the curve $y = \frac{5}{x^2}$ from 1 to 2 using ten approximating rectangles of equal widths and right endpoints.

a. 2.28  
b. 3.58  
c. 1.14  
d. 1.28  
e. 2.32  
f. 2.70
56 If \( f(x) = \frac{5}{\sqrt{x}} \), \( 1 \leq x \leq 4 \), approximate the area under the curve using ten approximating rectangles of equal widths and left endpoints.

a. 3.52  
b. 3.72  
c. 3.64  
d. 3.86  
e. 3.76  
f. 4.20

57 If \( f(x) = \sin \sin x \), \( 0 \leq x \leq \pi/2 \), approximate the area under the curve using ten approximating rectangles of equal widths and left endpoints.

a. 2.55  
b. 2.45  
c. 0.83  
d. 0.35  
e. 1.03  
f. 2.47

58 The speed of a runner increased steadily during the first three seconds of a race. Her speed at half-second intervals is given in the table. Find a lower estimate for the distance that she traveled during these three seconds.

<table>
<thead>
<tr>
<th>t (s)</th>
<th>0</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>v (ft/s)</td>
<td>0</td>
<td>5</td>
<td>9.8</td>
<td>14.8</td>
<td>16.5</td>
<td>17.3</td>
<td>17.7</td>
</tr>
</tbody>
</table>

a. 30.9  
b. 32.6  
c. 31.7  
d. 30.1  
e. 29.8  
f. 30.3
59 When we estimate distances from velocity data, it is sometimes necessary to use times \( t_0, t_1, t_2, \ldots \) that are not equally spaced. We can still estimate distances using the time periods \( \Delta t = t_i - t_{i-1} \). For example, on May 7, 1992, the space shuttle Endeavor was launched on mission STS – 49, the purpose of which was to install a new perigee kick motor in an Intelsat communications satellite. The table, provided by NASA, gives the velocity data for the shuttle between liftoff and the jettisoning of the solid rocket boosters. Use this data to estimate an upper bound for the space shuttle Endeavor's height above Earth's surface 61 seconds after liftoff.

<table>
<thead>
<tr>
<th>Event</th>
<th>Time (s)</th>
<th>Velocity (ft/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Launch</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Begin roll maneuver</td>
<td>9</td>
<td>185</td>
</tr>
<tr>
<td>End roll maneuver</td>
<td>15</td>
<td>309</td>
</tr>
<tr>
<td>Throttle to 89%</td>
<td>22</td>
<td>453</td>
</tr>
<tr>
<td>Throttle to 67%</td>
<td>33</td>
<td>748</td>
</tr>
<tr>
<td>Throttle to 104%</td>
<td>55</td>
<td>1329</td>
</tr>
<tr>
<td>Maximum dynamic pressure</td>
<td>61</td>
<td>1463</td>
</tr>
<tr>
<td>Solid rocket booster separation</td>
<td>122</td>
<td>4379</td>
</tr>
</tbody>
</table>

a. 52934  
b. 52536  
c. 52865  
d. 52105  
e. 53604  
f. 52745

60 The velocity graph of a braking car is shown. Use it to estimate to the nearest foot the distance traveled by the car while the brakes are applied. Use a left sum with \( n = 7 \).

<table>
<thead>
<tr>
<th>( t_i )</th>
<th>( \Delta t )</th>
<th>( v(t_i) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>0</td>
</tr>
</tbody>
</table>

a. 31  
b. 25  
c. 30  
d. 21  
e. 23  
f. 27
The velocity graph of a car accelerating from rest to a speed of 10 km/h over a period of 10 seconds is shown. Estimate to the nearest integer the distance traveled during this period. Use a right sum with \( n = 10 \).

\[ y = \tan x, \ 0 \leq x \leq \frac{\pi}{10} \]

\[ y = \tan x, \ 0 \leq x \leq \frac{\pi}{14} \]

\[ y = \tan x, \ 0 \leq x \leq \frac{\pi}{12} \]

\[ y = \tan x, \ 0 \leq x \leq \frac{\pi}{8} \]

\[ y = \tan x, \ 0 \leq x \leq \frac{\pi}{4} \]

\[ y = \tan x, \ 0 \leq x \leq \frac{\pi}{1} \]
Find an expression for the area from 4 to 8 under the curve $y = x^5$ as a limit.

a. $\lim_{n \to \infty} \sum_{i=1}^{n} \left( 4 + \frac{7i}{n} \right)^5 \frac{5}{n}$

b. $\lim_{n \to \infty} \sum_{i=1}^{n} \left( 4 + \frac{4i}{n} \right)^5 \frac{4}{n}$

c. $\lim_{n \to \infty} \sum_{i=1}^{n} \left( 4 + \frac{6i}{n} \right)^5 \frac{6}{n}$

d. $\lim_{n \to \infty} \sum_{i=1}^{n} \left( 4 + \frac{5i}{n} \right)^5 \frac{5}{n}$

e. $\lim_{n \to \infty} \sum_{i=1}^{n} \left( 4 + \frac{5i}{n} \right)^5 \frac{6}{n}$

f. $\lim_{n \to \infty} \sum_{i=1}^{n} \left( 4 + \frac{6i}{n} \right)^5 \frac{3}{n}$

Use a computer algebra system to find the area from 0 to 3 under the curve $y = x^5$.

a. 117.83
b. 121.36
c. 130.28
d. 113.45
e. 119.18
f. 121.50

Use a computer algebra system to find the exact area of the region from 0 to 3 under the graph of $y = e^{-x}$.

a. 0.52
b. 0.91
c. 0.82
d. 0.95
e. 0.46
f. 1.62
Use a computer algebra system to find the area of the region from 0 to $\frac{\pi}{3}$ under the cosine curve $y = \cos x$.

1. 1.65
2. 1.86
3. 1.78
4. 0.87
5. 1.40
6. 0.74

Evaluate the Riemann sum for $f(z) = 5 - z^2$, $0 \leq z \leq 2$, with four subintervals, taking the sample points to be right endpoints.

1. 6.25
2. 62.5
3. 1.5

If $f(z) = \sqrt{z} - 4$, $1 \leq z \leq 6$, find the Riemann sum with $n = 5$ correct to 3 decimal places, taking the sample points to be midpoints.

1. -3.789816
2. -10.856759
3. -21.713519
A table of values of an increasing function \( f(x) \) is shown. Use the table to find an upper estimate of \( \int_{0}^{25} f(r) \, dr \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>45</td>
</tr>
<tr>
<td>5</td>
<td>34</td>
</tr>
<tr>
<td>10</td>
<td>25</td>
</tr>
<tr>
<td>15</td>
<td>14</td>
</tr>
<tr>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>25</td>
<td>33</td>
</tr>
</tbody>
</table>

a. 450  
b. 180  
c. 150  
d. 540

The table gives the values of a function obtained from an experiment. Use the values to estimate \( \int_{0}^{6} f(t) \, dt \) using three equal subintervals with left endpoints.

<table>
<thead>
<tr>
<th>( t )</th>
<th>( f(t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>9.9</td>
</tr>
<tr>
<td>1</td>
<td>8.8</td>
</tr>
<tr>
<td>2</td>
<td>8.5</td>
</tr>
<tr>
<td>3</td>
<td>6.5</td>
</tr>
<tr>
<td>4</td>
<td>4.4</td>
</tr>
<tr>
<td>5</td>
<td>6.7</td>
</tr>
<tr>
<td>6</td>
<td>10.2</td>
</tr>
</tbody>
</table>

a. 16.6  
b. 45.6  
c. 37
Use the Midpoint Rule with \( n = 10 \) to approximate the integral.

\[
\int_{1}^{2} \sqrt{5 + r^2} \, dr
\]

a. 1.868562
b. 2.703176
c. 10.812706

Use the Midpoint Rule with \( n = 5 \) to approximate the integral.

\[
\int_{0}^{10} 10 \sin \sqrt{y} \, dy
\]

da. 23.907
b. 44.991
c. 64.643

Express the limit as a definite integral on the given interval.

\[
\lim_{x \to \infty} \sum_{i=1}^{n} 2 \, y_i \sin y_i \, \Delta y \, , \ [2,18]
\]

a. \( \int_{2}^{18} 2 \sin y \, dy \)
b. \( \int_{2}^{18} 2y \, dy \)
c. \( \int_{0}^{18} 2 \sin y \, dy \)

Express the limit as a definite integral on the given interval.

\[
\lim_{x \to \infty} \sum_{i=1}^{n} \left[ 3 \, q_i^2 - 15 \, q_i \right] \, \Delta q \, , \ [7,11]
\]

a. \( \int_{3}^{15} \left( 7 \, q^2 - 11 \, q \right) \, dq \)

b. \( \int_{7}^{11} \left( 3 \, q^2 + 15 \, q \right) \, dq \)

c. \( \int_{7}^{11} \left( 3 \, q^2 - 15 \, q \right) \, dq \)
75 Express the integral as a limit of sums. Then evaluate the limit.

\[ \int_{0}^{\pi} \sin 9x \, dx \]

a. \( \frac{2}{9} \)

b. \( \frac{1}{9} \)

c. \( \frac{\pi}{9} \)

76 Evaluate the integral by interpreting it in terms of areas.

\[ \int_{1}^{3} (3 + 2x) \, dx \]

a. 19

b. 10

c. 14

77 Evaluate the integral by interpreting it in terms of areas.

\[ \int_{-2}^{2} \sqrt{4 - x^2} \, dx \]

a. \( 4\pi \)

b. \( 8\pi \)

c. \( 2\pi \)

78 Evaluate the integral by interpreting it in terms of areas.

\[ \int_{-6}^{0} \left( 1 + \sqrt{36 - x^2} \right) \, dx \]

a. \( 18\pi + 6 \)

b. \( 9\pi + 6 \)

c. \( 9\pi - 6 \)
Evaluate the integral by interpreting it in terms of areas.

\[
\int_{-1}^{3} (2 - x) \, dx
\]

a. 5  
b. 4  
c. -8

Given that \( \int_{4}^{7} f(x) \, dx = \frac{2}{37} \), find \( \int_{7}^{4} \sqrt{t} \, dt \).

a. \( \frac{37}{2} \)  
b. \( -\frac{2}{37} \)

Express the sum as a single integral in the form \( \int_{a}^{b} f(q) \, dq \).

\[
\int_{2}^{9} f(q) \, dq + \int_{9}^{12} f(q) \, dq
\]

a. \( \int_{2}^{9} f(q) \, dq \)  
b. \( \int_{9}^{12} f(q) \, dq \)  
c. \( \int_{2}^{12} f(q) \, dq \)

If \( \int_{5}^{14} f(x) \, dx = 3.1 \) and \( \int_{6}^{14} f(x) \, dx = 0.7 \), find \( \int_{5}^{6} f(x) \, dx \).

a. -2.4  
b. 2.4  
c. 3.8
Find \( g'(x) \) by evaluating the integral using Part 2 of the Fundamental Theorem and then differentiating.

\[
g(x) = \int_\pi^x (3 + \cos(t)) \, dt
\]

\[
g(x) = \int_\pi^x (3 + \cos(t)) \, dt
\]

a. \( \frac{dg(x)}{dx} = 3x + \sin(x) \)

b. \( \frac{dg(x)}{dx} = 3 + \cos(x) \)

c. \( \frac{dg(x)}{dx} = -\sin(x) \)

Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

\[
g(x) = \int_1^x \sqrt{3 + 6t} \, dt
\]

a. \( \frac{dg(x)}{dx} = \frac{\sqrt{2 + 6x}}{2 \sqrt{3 + 6x}} \)

b. \( \frac{dg(x)}{dx} = \frac{\sqrt{3 + 6x}}{2 \sqrt{3 + 6x}} \)

c. \( \frac{dg(x)}{dx} = \frac{6}{2 \sqrt{3 + 6x}} \)

Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

\[
g(x) = \int_5^x \sin(t) \, dt
\]

a. \( \frac{dg(x)}{dx} = x^7 \sin(x) \)

b. \( \frac{dg(x)}{dx} = 6x^6 \sin(x) \)

c. \( \frac{dg(x)}{dx} = \frac{x^8}{8} \cos(x) \)
86 Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

\[ g(x) = \int_{x}^{7} 3\tan(t) \, dt \]

a. \( \frac{dg(x)}{dx} = 3\tan(x) \)

b. \( \frac{dg(x)}{dx} = -3\tan(x) \)

c. \( \frac{dg(x)}{dx} = 3\tan(7) \)

87 Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

\[ g(x) = \int_{9}^{2} \sqrt[3]{1 + t^6} \, dt \]

a. \( \frac{dg(x)}{dx} = 6x\sqrt[3]{1 + x^6} \)

b. \( \frac{dg(x)}{dx} = 3\sqrt[3]{1 + x^{12}} \)

c. \( \frac{dg(x)}{dx} = 6x\sqrt[3]{1 + x^{12}} \)

88 Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

\[ g(x) = \int_{9}^{\sqrt{x}} \frac{7 \cos(t)}{t} \, dt \]

a. \( \frac{dg(x)}{dx} = 3.5 \frac{\cos(\sqrt{x})}{\sqrt{x}} \)

b. \( \frac{dg(x)}{dx} = 7\cos(\sqrt{x}) \)

c. \( \frac{dg(x)}{dx} = 3.5 \frac{\cos(\sqrt{x})}{x} \)

89 Evaluate the integral. \( \int_{0}^{3} \left( 4 + 6y - y^2 \right) \, dy \)

a. 48

b. -24

c. 30

d. 39
Evaluate the integral. \( \int_{0}^{3} \frac{7}{x} \, dx \)

- a. \( \frac{10}{7} \)
- b. \( \frac{3}{10} \)
- c. \( \frac{7}{10} \)
- d. \( \frac{10}{3} \)

Evaluate the integral. \( \int_{0}^{4} \sqrt{x} \, dx \)

- a. \( \frac{8}{3} \)
- b. 4
- c. 8
- d. \( \frac{16}{3} \)

Evaluate the integral. \( \int_{\frac{\pi}{4}}^{\frac{3\pi}{2}} \cos \theta \, d\theta \)

- a. 0
- b. 1
- c. -1
- d. 2

Evaluate the integral. \( \int_{0}^{\frac{\pi}{2}} \sin \theta \, d\theta \)

- a. -0.318
- b. -0.159
- c. 0.318
- d. 0.159
- e. 0.079
94 Find the area of the region that lies beneath the given curve.

\[ y = \sin x, \quad 0 \leq x \leq \frac{\pi}{3} \]

a. 1.500  

b. -1.500  

c. 0.500  

d. -0.500

95 Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

\[ g(x) = \int \frac{4t^2 + 7}{5x^2 - 7} \, dt \]

a. \[ \frac{dg(x)}{dx} = \frac{25x^2 + 7}{25x^2 - 7} + \frac{16x^2 + 7}{16x^2 - 7} \]

b. \[ \frac{dg(x)}{dx} = -5 \frac{25x^2 + 7}{25x^2 - 7} + 4 \frac{16x^2 + 7}{16x^2 - 7} \]

c. \[ \frac{dg(x)}{dx} = -5 \frac{x^2 + 7}{x^2 - 7} + 4 \frac{x^2 + 7}{x^2 - 7} \]

96 Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

\[ g(x) = \int \cos(t^9) \, dt \]

a. \[ \frac{dg(x)}{dx} = 2\cos((2x)^9) - \sin(x)\cos(\cos^9(x)) \]

b. \[ \frac{dg(x)}{dx} = 2\cos((2x)^9) + \sin(x)\cos(\cos^9(x)) \]

c. \[ \frac{dg(x)}{dx} = \cos((2x)^9) + \sin(x)\cos(x) \]

97 If \( F(x) = \int_1^x f(t) \, dt \), where \( f(t) = \int_1^t \frac{\sqrt{9 + u^2}}{u} \, du \), find \( F''(2) \).

a. \( 2\sqrt{265} \)

b. \( \sqrt{265} \)

c. \( \frac{\sqrt{265}}{2} \)
98 Find the interval on which the curve is concave upward.

\[ F(x) = \int_{0}^{x} \frac{1}{5 + 2t} \, dt \]

a. \((-\infty, -\frac{5}{2})\)

b. \((-\frac{5}{2}, \infty)\)

c. \((-\infty, +\infty)\)

99 Let

\[ f(x) = \begin{cases} 
0 & \text{if } x < 0 \\
x & \text{if } 0 \leq x \leq 7 \\
14 - x & \text{if } 7 < x \leq 14 \\
0 & \text{if } x > 14 
\end{cases} \]

and

\[ g(x) = \int_{0}^{x} f(t) \, dt \]

Find an expression for \(g(x)\) when \(7 < x < 14\).

a. \(g(x) = 14x - \frac{1}{2}x^2 - 24.5\)

b. \(g(x) = 14x - \frac{1}{2}x^2 - 49\)

c. \(g(x) = 14x - \frac{1}{2}x^2\)

100 Find a function \(f(x)\) such that

\[ 8 + \int_{a}^{x} \frac{f(t)}{t^2} \, dt = 4\sqrt{x} \]

for \(x > 0\) and some \(a\).

a. \(f(x) = -\frac{5}{4x^2}\)

b. \(f(x) = \frac{3}{2x^2}\)

c. \(f(x) = \frac{4}{3}x^{\frac{1}{2}} - 8x\)
A high-tech company purchases a new computing system whose initial value is $V$. The system will depreciate at the rate $f(t)$ and will incur maintenance costs at the rate $g(t)$, where $t$ is the time measured in months. The company wants to determine the optimal time to replace the system.

Suppose that

$$f(t) = \begin{cases} \frac{V}{18} - \frac{Vt}{648} & \text{if } 0 < t \leq 46 \\ 0 & \text{if } t > 46 \end{cases}$$

Determine the length of time for the total depreciation $D(t) = \int_0^t f(s) \, ds$ to equal the initial value $V$.

a. $T = 648$
b. $T = 18$
c. $T = 36$

d. $x^2 + \frac{2}{7} x^7 + C$

e. $2 x^2 + \frac{2}{6} x^6 + C$

Find the general indefinite integral.

$$\int x \left( 4 + 2 x^5 \right) \, dx$$

a. $2 x^2 + 2 x^7 + C$
b. $4 x + \frac{2}{7} x^7 + C$
c. $2 x^2 + \frac{2}{7} x^7 + C$
d. $2 x^2 + \frac{2}{6} x^6 + C$
Find the general indefinite integral.

\[ \int (6 - t) \left( 6 + t^2 \right) \, dt \]

a. \[ 12t - 3t^2 + 2t^3 - \frac{t^4}{4} + C \]

b. \[ 36t + 2t^2 + 3t^3 - \frac{t^4}{4} + C \]

c. \[ 36t - 3t^2 + 2t^3 + \frac{t^4}{4} + C \]

d. \[ 36t - 3t^2 - 2t^3 - \frac{t^4}{4} + C \]

e. \[ 36t - 3t^2 + 2t^3 - \frac{t^4}{4} + C \]

f. \[ 36t - 3t^2 + 2t^3 - t^4 + C \]

Find the general indefinite integral.

\[ \int \frac{\sin 12t}{\sin 6t} \, dt \]

a. \[ -\frac{\sin 6t}{3} + C \]

b. \[ \frac{\sin 6t}{6} + C \]

c. \[ \frac{\cos 6t}{3} + C \]

d. \[ \frac{\sin 6t}{3} + C \]

e. \[ -\frac{\cos 6t}{3} + C \]

Evaluate the integral. \[ \int_4^9 \frac{x^2 + 6}{\sqrt{x}} \, dx \]

a. 434
b. 170
c. 217
d. 96.4
106 Evaluate the integral. \[ \int_{0}^{\pi/6} \frac{5 + \cos^2 \theta}{\cos^2 \theta} \, d\theta \]

a. 2.887  
b. 9.184  
c. 3.410  
d. 2.148

107 Evaluate the integral. \[ \int_{-2}^{4} \left( 3x - x^2 \right) \, dx \]

a. 51.00  
b. 99.00  
c. 63.00  
d. 15.00  
e. 54.00

108 The area of the region that lies to the right of the \( y \)-axis and to the left of the parabola \( x = 2y - y^2 \) (the shaded region in the figure) is given by the integral \[ \int_{0}^{2} \left( 2y - y^2 \right) \, dy. \]

Find the area of the region.

a. 8  
b. \( \frac{4}{6} \)  
c. 1  
d. \( \frac{8}{6} \)
109 If \( r' \) is a child's rate of growth in pounds per year, which of the following expressions represents the increase in the child's weight (in pounds) between the years 2 and 10?

a. \( r'(10) - r'(2) \)

b. \( \int_2^{10} r'(t) \, dt \)

110 Evaluate the integral. \( \int_1^4 x \, dx \)

a. \( \frac{244}{5} \)

b. \( \frac{80}{4} \)

c. \( \frac{26}{4} \)

d. \( \frac{242}{5} \)

e. 242

111 The velocity function (in meters per second) is given for a particle moving along a line. Find the distance traveled by the particle during the given time interval.

\[ v(t) = 6t - 5, \quad 0 \leq t \leq 4 \]

a. 68m

b. 28m

c. -8m

d. 94.75m

112 The acceleration function (in \( \text{m/s}^2 \)) and the initial velocity are given for a particle moving along a line. Find the velocity at time \( t \) and the distance traveled during the given time interval.

\[ a(t) = t + 5, \quad v(0) = 3, \quad 0 \leq t \leq 10 \]

a. \( v(t) = \frac{t^2}{2} + 5t + 3 \text{ m/s}, \quad 441 \frac{2}{3} \text{ m} \)

b. \( v(t) = \frac{t^2}{2} + 5t + 3 \text{ m/s}, \quad 446 \frac{2}{3} \text{ m} \)

c. \( v(t) = \frac{t^2}{2} + 5t \text{ m/s}, \quad 456 \frac{2}{3} \text{ m} \)

d. \( v(t) = \frac{t^2}{2} + 3 \text{ m/s}, \quad 461 \frac{2}{3} \text{ m} \)
An animal population is increasing at a rate of \( 20 + 48t \) per year (where \( t \) is measured in years). By how much does the animal population increase between the fourth and tenth years?

a. 4152  
b. 1896  
c. 2136  
d. 2036

The velocity of a car was read from its speedometer at ten-second intervals and recorded in the table. Use the Midpoint Rule to estimate the distance traveled by the car.

<table>
<thead>
<tr>
<th>( t ) (s)</th>
<th>( v ) (mi/h)</th>
<th>( t ) (s)</th>
<th>( v ) (mi/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>60</td>
<td>68</td>
</tr>
<tr>
<td>10</td>
<td>38</td>
<td>70</td>
<td>67</td>
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<tr>
<td>20</td>
<td>57</td>
<td>80</td>
<td>65</td>
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<td>30</td>
<td>69</td>
<td>90</td>
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<td>40</td>
<td>54</td>
<td>100</td>
<td>42</td>
</tr>
<tr>
<td>50</td>
<td>55</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. 1.6 miles  
b. 1.2 miles  
c. 1.5 miles  
d. 1.7 miles

The marginal cost of manufacturing \( x \) yards of a certain fabric is \( C'(x) = 3 - 0.01x + 0.000006x^2 \) (in dollars per yard). Find the increase in cost if the production level is raised from 1500 yards to 5000 yards.

a. $142000.00  
b. $141000.00  
c. $143000.00  
d. $140000.00

Evaluate the integral.

\[
\int \left( x^2 + 2 + \frac{1}{x^2 + 1} \right) \, dx
\]

a. \( \frac{x^3}{3} + 2x + \tan^{-1} x + c \)  
b. \( x^3 + 2 + \tan^{-1} x + c \)  
c. \( \frac{x^3}{3} + 2x + \frac{3}{x^3 + 3} + c \)
117 Evaluate the integral by making the given substitution:

\[ \int \cos 3x \, dx, \quad u = 3x \]

a. \( \frac{1}{3} \sin 3x \)
b. \( \frac{1}{3} \sin x + C \)
c. \( \frac{1}{3} \cos 3x + C \)
d. \( \sin 3x + C \)
e. \( \frac{1}{3} \sin 3x + C \)
f. \( -\frac{1}{3} \sin 3x + C \)

118 Evaluate the integral by making the given substitution:

\[ \int x^2 \sqrt{x^3 + 2} \, dx, \quad u = x^3 + 2 \]

a. \( -\frac{2}{9} \left( x^3 + 2 \right)^{\frac{3}{2}} + C \)
b. \( \frac{2}{9} \left( x^3 + 2 \right)^{\frac{3}{2}} \)
c. \( \frac{1}{9} \left( x^3 + 2 \right)^{\frac{1}{2}} + C \)
d. \( \frac{2}{9} \left( x^3 + 2 \right)^{\frac{3}{2}} + C \)
e. \( \frac{2}{9} \left( x^3 + 2 \right)^{\frac{1}{2}} + C \)
119 Evaluate the integral by making the given substitution:

\[ \int \frac{12}{(1 + 2x)^3} \, dx, \quad u = 1 + 2x \]

a. \( 3 \frac{1}{(1 + 2x)^2} + C \)

b. \( -6 \frac{1}{(1 + 2x)^2} + C \)

c. \( -3 \frac{1}{(1 + 2x)^4} \)

d. \( -3 \frac{1}{(1 + 2x)^2} + C \)

e. \( -3 \frac{1}{(1 + 2x)^4} + C \)

120 Evaluate the indefinite integral:

\[ \int 4x(x^2 + 6)^4 \, dx \]

a. \( (x^2 + 6)^5 + C \)

b. \( \frac{2}{5} (x^2 + 6)^3 + C \)

c. \( \frac{4}{5} (x^2 + 6)^5 + C \)

d. \( \frac{2}{5} (x^2 + 6)^3 + C \)

e. \( \frac{2}{5} (x^2 + 6)^4 + C \)

f. \( \frac{2}{5} (x^2 + 6)^5 + C \)
Evaluate the indefinite integral:
\[ \int \frac{3 + 18x}{\sqrt{3 + 3x + 9x^2}} \, dx \]

a. \[ -2\sqrt{3 + 3x + 9x^2} \]

b. \[ -2\sqrt{3 + 3x + 9x^2} + C \]

c. \[ \sqrt{3 + 3x + 9x^2} + C \]

d. \[ 3\sqrt{3 + 3x + 9x^2} + C \]

e. \[ 2\sqrt{3 + 3x + 9x^2} + C \]

Evaluate the indefinite integral:
\[ \int t^2 \cos \left(4 - t^3 \right) \, dt \]

a. \[ -\sin \left(4 - t^3 \right) + C \]

b. \[ \frac{1}{3} \sin \left(4 - t^3 \right) \]

c. \[ \frac{1}{3} \sin \left(4 - t^3 \right) + C \]

d. \[ -\frac{1}{3} \sin \left(4 - t^3 \right) + C \]

e. \[ -\frac{1}{2} \sin \left(4 - t^3 \right) + C \]

f. \[ \frac{1}{3} \cos \left(4 - t^3 \right) + C \]
123 Evaluate the indefinite integral:

\[ \int \cos^2 x \sin x \, dx \]

a. \[ -\frac{1}{3} \cos^2 x + C \]

b. \[ \frac{1}{3} \sin^2 x + C \]

c. \[ -\frac{1}{3} \cos^3 x + C \]

d. \[ -\frac{1}{3} \sin^3 x + C \]

e. \[ \frac{1}{3} \cos^3 x + C \]

f. \[ \frac{1}{3} \sin^3 x + C \]

124 Evaluate the definite integral:

\[ \int_0^1 x^2 \left( 2 + 4x^3 \right)^4 \, dx \]

a. \[ \frac{7744}{61} \]

b. \[ \frac{7744}{63} \]

c. \[ \frac{7744}{60} \]

d. \[ \frac{7749}{60} \]

e. \[ \frac{7744}{62} \]

f. \[ \frac{7747}{60} \]
Evaluate the definite integral:

\[ \int_{0}^{\pi/4} \sin 4t \, dt \]

a. 0  
b. 1.5  
c. 0.6  
d. 0.5  
e. −1.5  
f. 3.5

Find the area of the region that lies under the given curve:

\[ y = \sqrt{6x + 3}, \quad 0 \leq x \leq 1 \]

a. 2.423  
b. 2.523  
c. 2.403  
d. 2.426  
e. 2.533  
f. 2.298

If \( f \) is continuous and \( \int_{0}^{64} f(x) \, dx = 24 \), find \( \int_{0}^{8} f(8x) \, dx \).

a. 5  
b. 8  
c. 3  
d. 13  
e. 1  
f. −2
128 Evaluate the indefinite integral:

\[ \int \frac{e^x}{e^x + 8} \, dx \]

a. \( \ln(e^x - 8) + C \)

b. \( \ln(e^x + 8) + C \)

c. \( -\frac{1}{2} \ln(e^x + 8) + C \)

d. \( \frac{1}{2} \ln(e^x + 8) + C \)

e. \( -\ln(e^x + 8) + C \)

129 Evaluate the definite integral:

\[ \int_{e^6}^{64} \frac{dx}{x \sqrt{\ln x}} \]

a. 8.1

b. 9

c. 6

d. 7.5

e. 11

f. 8

130 Sketch the region enclosed by the curves \( y = x + 1 \), \( y = 25 - x^2 \), \( x = -1 \), and \( x = 2 \). Decide whether to integrate with respect to \( x \) or \( y \). Draw a typical approximating rectangle and label its height and width. Then find the area of the region.

a. 67.5

b. 16.88

c. 22.5

d. 135

e. 68.5

f. 202.5
Sketch the region enclosed by \( y = 10x + 6 \) and \( y = 4x^2 \). Decide whether to integrate with respect to \( x \) or \( y \). Draw a typical approximating rectangle and label its height and width. Then find the area of the region.

a. 30.58  
b. 9.53  
c. 7.15  
d. 114.33  
e. 85.75  
f. 28.58

Sketch the region enclosed by \( y = 10 + \sqrt{x} \) and \( y = \frac{6 + x}{10} \). Decide whether to integrate with respect to \( x \) or \( y \). Draw a typical approximating rectangle and label its height and width. Then find the area of the region.

a. 727.897728  
b. 1455.795455  
c. 4367.386366  
d. 2911.59091  
e. 485.265152  
f. 1456.795455

Sketch the region enclosed by \( y = 8x^2 \) and \( y = x^2 + 8 \). Decide whether to integrate with respect to \( x \) or \( y \). Draw a typical approximating rectangle and label its height and width. Then find the area of the region.

a. 57.015732  
b. 3.801049  
c. 11.403146  
d. 1.900524  
e. 13.403146  
f. 34.209439

Sketch the region enclosed by \( x = 7 - y^2 \) and \( x = y^2 - 4 \). Decide whether to integrate with respect to \( x \) or \( y \). Draw a typical approximating rectangle and label its height and width. Then find the area of the region.

a. 8.599096  
b. 5.73273  
c. 34.396382  
d. 103.189147  
e. 137.585529  
f. 36.396382
135 Use calculus to find the area of the triangle with the given vertices.

\[(0, 0), (11, 1), (-1, 15)\]

a. \(S = 83.5\)
b. \(S = 82\)
c. \(S = 81.5\)
d. \(S = 83\)

136 Use the Midpoint Rule with \(n = 4\) to approximate the area of the region bounded by the given curves.

\[y = \sqrt{1 + x^3}, y = 1 - 2x, x = 2\]

a. \(S = 7.22\)
b. \(S = 5.22\)
c. \(S = 6.22\)
d. \(S = 8.22\)

137 Use a graph to find approximate \(x\) – coordinates of the points of interception of the curves. Then find (approximately) the area of the region bounded by the curves.

\[y = 2\cos x, \ y = x^4\]

a. \(A \approx 2.97\)
b. \(A \approx 3.83\)
c. \(A \approx 1.48\)
d. \(A \approx 2.23\)
Racing cars driven by Chris and Kelly are side by side at the start of a race. The table shows the velocities of each car (in miles per hour) during the first ten seconds of the race. Use the Midpoint Rule to estimate how much farther Kelly travels than Chris does during the ten seconds.

<table>
<thead>
<tr>
<th>t</th>
<th>V_C</th>
<th>V_K</th>
<th>t</th>
<th>V_C</th>
<th>V_K</th>
</tr>
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<td>0</td>
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<td>82</td>
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<td>10</td>
<td>90</td>
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<tr>
<td>5</td>
<td>65</td>
<td>74</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. \( \frac{117}{3} \) ft
b. \( \frac{118}{3} \) ft
c. \( \frac{115}{3} \) ft
d. \( \frac{120}{3} \) ft

Find the area of the region bounded by the parabola \( y = x^2 \), the tangent line to this parabola at \((4, 16)\), and the \( x \)-axis.

a. 23.333333
b. 21.333333
c. 25.333333
d. 26.333333
e. 22.333333

Find the number \( b \) such that the line \( y = b \) divides the region bounded by the curves \( y = 4x^2 \) and \( y = 1 \) into two regions with equal area.

a. \( \frac{4}{2^{3/2}} \)
b. \( \frac{1}{2^{3/2}} \)
c. \( \frac{1}{2^{3/2}} \)
d. \( \frac{4}{2^{3/2}} \)
e. \( \frac{1}{2^{3/2}} \)
f. \( \frac{4}{2^{3/2}} \)
Find the positive value of $c$ such that the area of the region bounded by the parabolas $y = x^2 - c^2$ and $y = c^2 - x^2$ is 576.

a. 4  
b. 16  
c. 5  
d. 6  
e. 8

Find the volume of the solid obtained by rotating about the $x-$axis the region under the curve $y = \frac{1}{x}$ from $x = 3$ to $x = 5$.

a. $\frac{8\pi}{15}$  
b. $\frac{2\pi}{15}$  
c. $8\pi$

Find the volume of the solid obtained by rotating the region in the first quadrant bounded by $y = x^2$ and $y = 3$ about the $y-$axis.

a. $\frac{9\pi}{2}$  
b. $\frac{9}{2}$  
c. $\frac{3\pi}{2}$

Find the volume of the solid obtained by rotating the region bounded by $y = x^4$ and $x = y^4$ about the $x-$axis.

a. $\frac{10\pi}{3}$  
b. $\frac{5\pi}{9}$  
c. $\frac{16\pi}{27}$
Find the volume of the solid obtained by rotating the region bounded by \( x = y^2 \) and \( x = 3y \) about the \( y \)-axis.

a. \( \frac{243}{15} \pi \)

b. \( \frac{1458}{15} \pi \)

c. \( -\frac{243}{3} \pi \)

Find the volume of the solid obtained by rotating the region bounded by \( y = \frac{3}{\sqrt{x}} \) and \( y = x \) about the line \( y = 1 \).

a. \( \frac{6}{15} \pi \)

b. \( \frac{6}{15} \)

c. \( \frac{4}{15} \)

d. \( \frac{4}{15} \pi \)

Find the volume of the solid obtained by rotating the region bounded by \( y = x^3 \) and \( x = y^3 \) about the line \( x = -1 \).

a. \( \frac{204}{20} \pi \)

b. \( \frac{219}{35} \pi \)

c. \( \frac{204}{140} \pi \)

Find the volume of a right circular cone with height \( h = 6 \) and base radius \( r = 9 \).

a. \( 162 \pi \)

b. \( 486 \pi \)

c. \( 162 \pi \)
149 Find the volume of a cap of a sphere with radius $r = 8$ and height $h = 0.75$.

$$V = \frac{1}{6} \pi h (3r^2 + h^2)$$

a. $4.5 \pi$

b. $4.359375 \pi$

c. $5.8125 \pi$

150 Find the volume of a pyramid with height $h = 27$ and rectangular base with dimensions 10 and 20.

$$V = \frac{1}{3} Bh$$

a. 180

b. 900

c. 1800

151 Find the volume of a pyramid with height 6 and base an equilateral triangle with side $a = 10$.  

$$V = \frac{1}{3} B h$$

a. 8.66

b. 86.6

c. 346.41

d. 173.21
152 Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the $y$-axis.

$$y = \frac{1}{x}, \ y = 0, \ x = 1, \ x = 9$$

a. $V = 8\pi$

b. $V = 7\pi$

c. $V = 17\pi$

b. $V = 16\pi$

153 Use the method of cylindrical shells to find the volume of solid obtained by rotating the region bounded by the given curves about the $x$-axis

$$x = 6 + y^2, \ x = 0, \ y = 1, \ y = 3$$

a. $V = 93\pi$

b. $V = 176\pi$

c. $V = 86\pi$

b. $V = 88\pi$

154 Use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by the given curves about the $x$-axis.

$$y^2 - 18y + x = 0, \ x = 0$$

a. $V = 17476\pi$

b. $V = 34992\pi$

c. $V = 17546\pi$

d. $V = 17496\pi$

155 Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the specified axis.

$$y = x^2, \ y = 0, \ x = 1, \ x = 8; \ \text{about} \ x = 1$$

a. $V = 20482\pi$

b. $V = 10246\pi$

c. $V = 10239\pi$

d. $V = \frac{10241}{6}\pi$
Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the specified axis.

\[ y = \sqrt{x - 1}, \ y = 0, \ x = 5; \ \text{about } y = 3 \]

a. \( V = 29\pi \)

b. \( V = 48\pi \)

c. \( V = 24\pi \)

d. \( V = 22\pi \)

Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis.

\[ y = \sin x, \ y = 0, \ x = 4\pi, \ x = 9\pi; \ \text{about the } y - \text{axis.} \]

a. \( V = \int_{4\pi}^{9\pi} 2x \sin(x) \, dx \)

b. \( V = \int_{4\pi}^{9\pi} 2\pi \sin(x) \, dx \)

c. \( V = \int_{0}^{4\pi} 2\pi x \sin(x) \, dx \)

d. \( V = \int_{4\pi}^{9\pi} 2\pi x \sin(x) \, dx \)

Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis.

\[ x = \sqrt{\sin y}, \ 0 \leq y \leq \pi, \ x = 0; \ \text{about } y = 4. \]

a. \( V = \int_{0}^{\pi} 2(4 - y) \sqrt{\sin y} \, dy \)

b. \( V = \int_{0}^{\pi} (4 - y) \sqrt{\sin y} \, dy \)

c. \( V = \int_{0}^{\pi} (4 - y) \sqrt{\sin y} \, dy \)

d. \( V = \int_{0}^{\pi} 2\pi (4 - y) \sqrt{\sin y} \, dy \)
Use the Midpoint Rule with \( n = 4 \) to estimate the volume obtained by rotating about the \( y \) -axis the region under the curve

\[ y = \tan x, \quad 0 \leq x \leq \frac{\pi}{4}. \]

a. \( V = 1.460 \)
b. \( V = 1.142 \)
c. \( V = 1.722 \)
d. \( V = 0.455 \)

Sketch a graph to estimate the \( x \) -coordinates of the points of intersection of the given curves. Then use this information to estimate the volume of the solid obtained by rotating about the \( y \) -axis the region enclosed by these curves. Please round the answers to the nearest hundredth.

\[ y = 0, \quad y = -x^4 + 6x^3 - x^2 + 6x \]

a. \( V = 3346.96 \pi \)
b. \( V = 3326.40 \pi \)
c. \( V = 3331.63 \pi \)
d. \( V = 3323.88 \pi \)

The region bounded by the given curves is rotated about the specified axis. Find the volume of the resulting solid by any method.

\[ y = x^2 - 2x - 3; \quad \text{about the} \ x \ \text{axis.} \]

a. \( V = 54.47 \pi \)
b. \( V = 39.58 \pi \)
c. \( V = 31.63 \pi \)
d. \( V = 34.13 \pi \)

The region bounded by the given curves is rotated about the specified axis. Find the volume of the resulting solid by any method.

\[ y = 5, \quad y = x^2 - 4x + 8; \quad \text{about} \ x = -1 \]

a. \( V = 13.45 \pi \)
b. \( V = 28.34 \pi \)
c. \( V = 8.00 \pi \)
d. \( V = 5.50 \pi \)
163 Use cylindrical shells to find the volume of the solid.
A sphere of radius $a$.

a. $V = \frac{2}{3} \pi a^3$

b. $V = \frac{1}{3} \pi a^3$

c. $V = \frac{4}{3} \pi a^3$

d. $V = \frac{5}{3} \pi a^3$

164 Use cylindrical shells to find the volume of the solid.
A right circular cone with height $f$ and base radius $t$.

a. $V = \frac{\pi ft^2}{3}$

b. $V = \frac{\pi ft^2}{2}$

c. $V = \frac{\pi ft^3}{3}$

d. $V = \frac{\pi ft^2}{3}$

165 Suppose you make napkin rings by drilling holes with different diameters through two wooden balls (which also have different diameters). You discover that both napkin rings have the same height $h$ as shown in the figure.
Use cylindrical shells to compute the volume of a napkin ring created by drilling a hole with radius $r$ through the center of a sphere of radius $D$ and express the answer in terms of $h$.

![](image)

a. $V = \frac{1}{6} \pi h^2$

b. $V = \frac{1}{4} \pi h^3$

c. $V = \frac{1}{6} \pi h^3$

d. $V = \frac{1}{3} \pi h^2$
Find the work done in pushing a car a distance of 12 m while exerting a constant force of 200 N.

a. \( W = 3900 \text{ J} \)
b. \( W = 2900 \text{ J} \)
c. \( W = 2700 \text{ J} \)
d. \( W = 2400 \text{ J} \)

A particle is moved along the \( x \)-axis by a force that measures \( \frac{4}{(1 + x)^2} \) pounds at a point \( x \) feet from the origin. Find the work done in moving the particle from the origin to a distance of 3 ft.

a. 0 lb - ft
b. 14 lb - ft
c. 5 lb - ft
d. -3 lb - ft
e. 3 lb - ft

If 384 J of work are needed to stretch a spring from 7 cm to 13 cm and another 672 J are needed to stretch it from 13 cm to 19 cm, what is the natural length of the spring?

a. 0
b. 2
c. 3
d. 1

d. 110 ft high. How much work is done in pulling the rope to the top of the building?

a. 251 ft - lb
b. 99 ft - lb
c. 200 ft - lb
d. 250 ft - lb
e. 350 ft - lb
A bucket that weighs 4 lb and a rope of negligible weight are used to draw water from a well that is 40 ft deep. The bucket starts with 35 lb of water and is pulled up at a rate of 10 ft/s, but water leaks out of a hole in the bucket at a rate of 0.1 lb/s. Find the work done in pulling the bucket to the top of the well.

a. 1552 ft\cdot lb
b. 1553 ft\cdot lb
c. 1502 ft\cdot lb
d. 1662 ft\cdot lb

An aquarium 3 m long, 8 m wide, and 1 m deep is full of water. Find the work needed to pump half of the water out of the aquarium. (Use the facts that the density of water is \(1000 \text{ kg/m}^3\) and \(g \approx 9.8 \text{ m/s}^2\).)

a. 29410 J
b. 29411 J
c. 29250 J
d. 28400 J
e. 29400 J

A tank is full of water. Find the work required to pump the water out of the outlet. Round the answer to the nearest thousand.

\[ h = 2 \text{ m}, \ r = 1 \text{ m}, \ d = 5 \text{ m} \]

a. \( W = 339000 \text{ J} \)
b. \( W = 184000 \text{ J} \)
c. \( W = 13000 \text{ J} \)
d. \( W = 462000 \text{ J} \)
173 The tank shown is full of water. Given that water weighs 62.5 lb/ft and \( R = 8 \), find the work required to pump the water out of the tank.

\[
P = P(V)
\]

\[
F = r^2 P.
\]

\[
W = V_2 Pd
\]

\[
W = V_1 Pd
\]

\[
W = V_2 Pd
\]

\[
W = V_1 Pd
\]

\[
W = V_2 Pd
\]

\[
W = V_1 Pd
\]

The tank shown is full of water. Given that water weighs 62.5 lb/ft and \( R = 8 \), find the work required to pump the water out of the tank.

- a. 200952 ft - lb
- b. 201062 ft - lb
- c. 200062 ft - lb
- d. 201082 ft - lb
- e. 200061 ft - lb

174 When gas expands in a cylinder with radius \( r \), the pressure at any given time is a function of the volume: \( P = P(V) \). The force exerted by the gas on the piston (see the figure) is the product of the pressure and the area: \( F = \pi r^2 P \). Find the work done by the gas when the volume expands from volume \( V_1 \) to volume \( V_2 \).

\[
W = \int VdP
\]

\[
W = \int VdP
\]

\[
W = \int PdV
\]

\[
W = \int PdV
\]

\[
W = \int VdP
\]
In a steam engine the pressure and volume of steam satisfy the equation $PV^{1.4} = k$, where $k$ is a constant. (This is true for adiabatic expansion, that is, expansion in which there is no heat transfer between the cylinder and its surroundings.) Calculate the work done by the engine during a cycle when the steam starts at a pressure of 110 lb/in$^2$ and a volume of 200 in$^3$ and expands to a volume of 1200 in$^3$.

Use the fact that the work done by the gas when the volume expands from volume $V_1$ to volume $V_2$ is

$$ W = \int_{V_1}^{V_2} P \, dV $$

a. 2346 ft$\cdot$lb
b. 1346 ft$\cdot$lb
c. 2455 ft$\cdot$lb
d. 2295 ft$\cdot$lb
e. 2345 ft$\cdot$lb

Newton's Law of Gravitation states that two bodies with masses $m_1$ and $m_2$ attract each other with a force

$$ F = G \frac{m_1 m_2}{r^2} $$

where $r$ is the distance between the bodies and $G$ is the gravitation constant. If one of the bodies is fixed, find the work needed to move the other from $r = k$ to $r = d$.

a. $W = Gm_1 m_2 \left( \frac{1}{d^2} - \frac{1}{k^2} \right)$
b. $W = Gm_1 m_2 \left( \frac{1}{d} - \frac{1}{k} \right)$
c. $W = Gm_1 m_2 \left( \frac{1}{k^2} - \frac{1}{d^2} \right)$
d. $W = Gm_1 m_2 \left( \frac{1}{k} - \frac{1}{d} \right)$

Find the average value of the function $z(t) = 5 \cos t$ on the interval $\left[ 0, \frac{\pi}{2} \right]$.

a. $\frac{20}{\pi}$
b. $\frac{5}{\pi}$
c. $\frac{10}{\pi}$
178 Find the average value of the function \( y(t) = 2\sqrt{t} \) on the interval \([1, 16]\).

a. \( \frac{756}{15} \)

b. \( \frac{252}{15} \)

c. \( \frac{252}{45} \)

d. \( \frac{252}{135} \)

179 Find a number \( a \in [0, 10] \) at which the value of the function \( f(t) = 7 - t^2 \) is equal to the average value of the function on the interval \([0, 10]\).

a. \( a = \frac{10\sqrt{3}}{3} \)

b. \( a = -\frac{10\sqrt{3}}{3} \)

c. \( a = \frac{10\sqrt{3}}{3} \)

d. \( a = \frac{20\sqrt{3}}{3} \)

180 Find the average value of the function \( u(x) = 10 \sin (x^2) \) on the interval \([0, \sqrt{\pi}] \).

a. \( \frac{20}{\sqrt{\pi}} \)

b. \( \frac{20}{\pi} \)

c. \( \frac{10}{\sqrt{\pi}} \)

d. \( \frac{10}{\pi} \)

181 Find the number(s) \( a \) such that the average value of the function \( f(x) = 21 - 14x + 3x^2 \) on the interval \([0, a]\) is equal to 9.

a. \( -3 \)

b. \( 8 \)

c. \( 4 \)

d. \( 3 \)
In a certain city the temperature $x$ hours after 8 A.M. was modeled by the function

$$U(x) = 35 + 14 \sin \frac{\pi x}{10}$$

Find the average temperature during the period from 8 A.M. to 8 P.M.

a. 32.003  
b. 41.718  
c. 45.574  
d. 23.524

The velocity $v$ of blood that flows in a blood vessel with radius $R$ and length $l$ at a distance $r$ from the central axis is

$$v(r) = \frac{P}{4q l} \left( R^2 - r^2 \right)$$

where $P$ is the pressure difference between the ends of the vessel and $q$ is the viscosity of the blood.

Suppose that Vessel 1 has length 1.6 cm and outer radius 1.2 mm, and Vessel 2 has length 4.8 cm and outer radius 4.5 mm. Find the average velocities (with respect to $r$) over the interval $0 \leq r \leq R$ for each vessel. Which vessel has the higher average velocity?

a. Vessel 2  
b. Vessel 1
|   | 1.   | 2.   | 3.   | 4.   | 5.   | 6.   | 7.   | 8.   | 9.   | 10.  | 11.  | 12.  | 13.  | 14.  | 15.  | 16.  | 17.  | 18.  | 19.  | 20.  | 21.  | 22.  | 23.  |
|---|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
|   | c    | f    | a    | e    | f    | e    | f    | d    | b    | e    | a    | c    | e    | e    | d    | c    | f    | d    | b    | a    | c    |
|   | 24. b| 47. a| 70. b| 93. d| 116. a| 139. b| 162. c|      |      |      |      |      |      |      |      |      |      |      |      |      |      |
|   | 25. a| 48. b| 71. b| 94. c| 117. e| 140. b| 163. c|      |      |      |      |      |      |      |      |      |      |      |      |      |      |
|   | a    | 26. a,b,e| 49. c| 72. c| 95. b| 118. d| 141. d| 164. d|      |      |      |      |      |      |      |      |      |      |      |      |      |
|   | e    | 27. a,d| 50. b| 73. a| 96. b| 119. d| 142. b| 165. c|      |      |      |      |      |      |      |      |      |      |      |      |      |
|   | f    | 28. e| 51. c| 74. c| 97. b| 120. f| 143. a| 166. d|      |      |      |      |      |      |      |      |      |      |      |      |      |
|   | e    | 29. b| 52. e| 75. a| 98. b| 121. e| 144. b| 167. e|      |      |      |      |      |      |      |      |      |      |      |      |      |
|   | f    | 30. c,e| 53. f| 76. c| 99. a| 122. d| 145. b| 168. b|      |      |      |      |      |      |      |      |      |      |      |      |      |
|   | d    | 31. b| 54. c| 77. c| 100. b| 123. c| 146. d| 169. d|      |      |      |      |      |      |      |      |      |      |      |      |      |
|   | e    | 32. c| 55. e| 78. b| 101. c| 124. c| 147. c| 170. a|      |      |      |      |      |      |      |      |      |      |      |      |      |
|   | b    | 33. c| 56. a| 79. b| 102. c| 125. d| 148. a| 171. e|      |      |      |      |      |      |      |      |      |      |      |      |      |
|   | a    | 34. c| 57. c| 80. b| 103. e| 126. a| 149. b| 172. d|      |      |      |      |      |      |      |      |      |      |      |      |      |
|   | c    | 35. b| 58. c| 81. c| 104. d| 127. c| 150. c| 173. b|      |      |      |      |      |      |      |      |      |      |      |      |      |
|   | d    | 36. c| 59. a| 82. b| 105. d| 128. b| 151. b| 174. c|      |      |      |      |      |      |      |      |      |      |      |      |      |
|   | b    | 37. c| 60. b| 83. b| 106. c| 129. f| 152. d| 175. e|      |      |      |      |      |      |      |      |      |      |      |      |      |
|   | a    | 38. c| 61. c| 84. b| 107. d| 130. a| 153. d| 176. d|      |      |      |      |      |      |      |      |      |      |      |      |      |
|   | c    | 39. c| 62. e| 85. a| 108. d| 131. f| 154. d| 177. c|      |      |      |      |      |      |      |      |      |      |      |      |      |
|   | f    | 40. d| 63. b| 86. b| 109. b| 132. b| 155. d| 178. c|      |      |      |      |      |      |      |      |      |      |      |      |      |
|   | d    | 41. c| 64. f| 87. c| 110. d| 133. c| 156. c| 179. c|      |      |      |      |      |      |      |      |      |      |      |      |      |
|   | a    | 42. b| 65. d| 88. c| 111. b| 134. c| 157. d| 180. c|      |      |      |      |      |      |      |      |      |      |      |      |      |
|   | c    | 43. b| 66. d| 89. c| 112. b| 135. d| 158. d| 181. c,d|      |      |      |      |      |      |      |      |      |      |      |      |      |
|   | a    | 44. b| 67. a| 90. c| 113. c| 136. b| 159. b| 182. b|      |      |      |      |      |      |      |      |      |      |      |      |      |
|   | c    | 45. c| 68. b| 91. d| 114. c| 137. a| 160. b| 183. a|      |      |      |      |      |      |      |      |      |      |      |      |      |
|   | d    | 46. b| 69. c| 92. a| 115. d| 138. a| 161. d|      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |