Math 502 - Complex Analysis
Course Topics - Spring 2009

Textbook:
[SS] Elias Stein, Rami Shakarchi, *Complex Analysis*, Princeton University Press 2003.

Reference Texts:
[A] Lars Ahlfors, *Complex Analysis. Third Edition*, McGraw – Hill 1979
[R] Walter Rudin, *Real and complex analysis. Third edition*. McGraw-Hill, 1987.

Course topics:
Topics in part **1) – 6)** are mandatory and form the material for the Qualifying Exam in Analysis – Part B. Topics in part **7)** will be covered depending on time.

1) Basics :

(a) Geometric description of complex numbers, the complex plane, point at infinity, Riemann sphere.

(b) Definition of conformality, linear transformations as conformal maps and representation by complex numbers.

(2 lectures, [SS] Chapter 1, Section 1, [A] Chapter 1.)

2) Holomorphic functions:

(a) Definition, examples (exponential and trigonometric functions).

(b) Holomorphic functions and conformality, Cauchy-Riemann equations, relation to harmonic functions.

(c) Power series: examples (exponential and trig functions), review of results on uniform convergence (Weierstrass M-test, continuity, integrability, differentiability).

(d) Review of integration along curves, primitives.

(5 lectures, [SS] Chapter 1, Section 2, 3.)

3) Cauchy's Theorem and Applications:

(a) Goursat's theorem, Cauchy theorem in a disk and other "toy domains" (e.g. rectangles, sectors, puctured disk), evaluation of integrals. Morera's theorem.

(b) Cauchy integral formulas, Cauchy estimates, Liouville's theorem, fundamental theorem of algebra. isolated zeroes and analytic continuation.

(c) Sequences of holomorphic functions. Schwarz Reflection Principle.

(5 lectures, Chapter 2 of [SS], excluding Sections 5.3 and 5.5.) [A] Chapter 4 Sections 1, 2).

4) **Meromorphic functions**:

   (a) Zeroes and poles, Laurent series. The residue formula for toy domains, Jordan and "Small Arc" lemmas, computation of integrals by residue calculus.
   (b) Riemann's theorem on removable singularities, essential singularities and Casorati–Weierstrass theorem.
   (c) The Argument Principle and applications (Rouche', Open Mapping, Maximum Modulus).
   (d) Definition of the Gamma and Zeta functions.

   (8 lectures, [SS] Chapter 3, Sections 1–1, Chapter 6 Sections 1.1, and 2.1.).

5) **Plane topology**:

   (a) Simply and multiply connected domains, Jordan curves (no proof), homotopies, the winding number and the general form of Cauchy's Theorem.
   (b) Roots and logarithm (including branches and cuts).
   (c) Additional examples of integral evaluation using residues.

   (5 lectures, [SS] Chapter 3, Section 5 – 7, [A], Chapter 3, Section 4 – 5)

6) **Conformal maps**:

   (a) Conformal equivalence and examples of elementary conformal maps.
   (b) Schwarz Lemma, automorphisms of the disk and upper-half plane, fractional linear transformations (cross ratio, behavior of lines and circles).
   (c) Definiton of normal family, Montel's Theorem and proof of the Riemann Mapping Theorem.

   (7 lectures, Chapter 8 of [SS], excluding Section 1.3 and Section 4, [A] Chapter 3, Section 3, 4).

7) **Possible additional topics**:

   (a) Fourier Transform: Paley-Wiener and functions of exponential type.
   (b) The Prime Number theorem.
   (c) Entire functions and Picard's theorem. Riemann surfaces.
   (d) Elliptic functions.