# Math 501 - Real Analysis
# Course Topics - Fall 2009

**Textbook (required):**
[Ro] H. L. Royden, *Real Analysis, Third Edition*, Prentice Hall, 1989.

Excellent reference texts are:
[F] G. B. Folland, *Real Analysis. Second Edition*, Wiley 1999,
[T] M. E. Taylor, *Measure Theory and Integration*, AMS, 2006,
[Ru] W. Rudin, *Real and complex analysis. Third edition.* McGraw-Hill, 1987.

**Course topics:** Topics in part **1)** − **6)** are mandatory and form the material for the Qualifying Exam in Analysis, Part A. Topics in part **7)** will be covered time-permitting.

**1) Metric spaces**: Metrics. Open and closed sets. Examples, including Cantor sets. Convergence and completeness. Subspaces. Baire category. ([Ro], Chapter 7, [F] Section 1.5.)

**2) Continuity**: Continuous functions and homeomorphisms. Uniform continuity. Contraction mapping principle. ([Ro], Chapter 7.)

**3) Topological spaces, compact spaces**: Basic properties and examples. Connectedness, path connectedness. Sequential compactness and covering compactness. Lebesgue covering theorem. Product of compact spaces. Locally compact spaces. The Stone-Weierstrass theorem. ([Ro], Chapters 8 and 9.)

**4) Lebesgue measure, sets of measure zero**: Construction of Lebesgue measure on $\mathbb{R}^n$. Outer measure, measurable sets and Lebesgue measure. Non-measurable sets. Additivity and continuity properties. Approximation by open and closed sets. Measurable functions. Lusins theorem. ([Ro], Chapter 3, [T], Chapter 2.)

**5) Integration and differentiation**: Construction of the Lebesgue integral. Basic properties. Fatous lemma. Monotone convergence theorem. Dominated convergence theorem. Differentiation of monotone functions. Functions of bounded variation. Differentiating under the integral sign. Absolute continuity. The Radon-Nikodym theorem. ([Ro], Chapter 4 and 5, parts of Chapter 11, [F] Chapter 3.)

**6) $L^p$ spaces**: Measure spaces and measurable functions. The spaces $L^1$ and $L^2$, including completeness. Fubinis theorem on $\mathbb{R}^n$. Brief discussion of product measures and the general case. Convolutions and the smoothing properties of convolution. ([Ro], Chapter 6, [$F$] Sections 6.1–6.3, [T], Chapter 4.)

**7) Possible additional topics**:

   (a) Fourier Transform: the inversion formula, Plancherel Theorem. ([F], Chapter 8.)

   (b) Hausdorff measure and Hausdorff dimension. Fractal sets.([F], Chapter 11, [T], Chapter 12.)

(c) Probability spaces and Brownian motion. ([T], Chapter 16 and 16.)