

Math 312, Fall 2004
Practice midterm 1
Total 40 pts+ 10 extra credit
Please, show the details of your work.
Put your name on the exam.
Good luck!

1a. Give the definition of an increasing sequence.

1b. Formulate the negative statement without using "not":
Sequence $\{a_n\}$ converges to zero.

2. State the uniqueness theorem for limits and prove it.

3. Evaluate the limits and prove you are correct:

$$a) a_{n+1} = a_n^2, a_1 = -1/2, n \in \mathbb{N},$$

$$b) b_n = 1 - \frac{1}{2} + \frac{1}{3} + \cdots + (-1)^n \frac{1}{n+1}, n \in \mathbb{N},$$

$$c) c_n = -\frac{n}{n^2 + 1}, n \in \mathbb{N}.$$

4. Suppose $a_n \rightarrow 0$ as $n \rightarrow \infty$, b_n is bounded. Prove that for any sequence $\{c_n\}$

$$\frac{a_n b_n}{1 + c_n^2} \rightarrow 0,$$

as $n \rightarrow \infty$.

5 (Extra credit). If $a_n \rightarrow L$, and b_n lies between a_n and a_{n+1} , prove $b_n \rightarrow L$.

Remark: Note that "between" does not tell you in which direction the inequalities go.