

REVIEW PROBLEMS - MIDTERM II

1) Consider the sequence of positive numbers defined by:

$$a_1 = 6 \quad a_n = \sqrt{6 + a_{n-1}} \quad n > 1$$

a) Show  $(a_n)$  monotone

b) Show  $(a_n)$  converges.

c) Find  $\lim_{n \rightarrow +\infty} a_n = a$

6) Find sum of the series:

$$\sum_{n=1}^{+\infty} \frac{6^n}{2^n}$$

2) Let  $\sum a_n$  be a convergent series. Show that  $\sum (a_n - a_{n+1})$  is also convergent.

3) Find all subsequential limits of

$$S_n = [(-1)^n + 1] = n^2, \quad S_n = (-1)^n \left(2 + \frac{3}{n}\right)$$

find  $\limsup$  &  $\liminf$

4) Let  $s_0 = 1$ ,  $s_2 = 1$ ,  $s_{n+1} = \frac{s_n + s_{n-1}}{2}$ ,  $n \geq 1$ .

Use Cauchy criterion to show  $(s_n)$  converges.

5) Determine whether  $\sum \frac{1}{n^2 \left[1 + \frac{1}{2} \sin\left(\frac{n\pi}{4}\right)\right]}$  converges or diverges.