Math 110: Review Problems for Midterm 1

The following represent select review problems on material covered prior to Midterm 1.

1. Find the domain of $\dfrac{1}{|x|\sqrt{1-x^2}}$.

   Ans:$(-1,1), x \neq 0$.

2. Describe the domain of the function $f(x) = \dfrac{1}{\sqrt{x+1}-\sqrt{x}}$, using interval notation.

   Ans: $[0,\infty)$

3. Suppose $-2 < 4-3x \leq 1$. Determine the largest interval to which x can belong.

   Ans: $[1,2)$.

4. Simplify $\left(\dfrac{\frac{x}{\sqrt{2}}}{\frac{1-x}{4}}\right)$.

   Ans: $\frac{2\sqrt{2}x}{1-x}$, after rationalizing the denominator.

5. Decide which is larger $-\sqrt{2} = -1.41421\ldots$ or $-1\dfrac{41}{99}$.

   Ans: $-1\frac{44}{99} = -1.4141\cdots > -1.4142\cdots = -\sqrt{2}$.

6. Consider the straight line which is perpendicular to the line $x+y=1$ and passes through the point $(1,1)$.

   (a) Where does this line intersect the $x-$axis?
   (b) Where does this line intersect the $y-$axis?
   (c) What is its slope?

   Ans: The equation of the line is $y=x$. Its x and y intercept is $(0,0)$ and its slope is 1.

7. Find the equation of the straight line passing through $(0,1)$ which descends 2 units for every unit that it moves to the right.

   Ans: $2x+y-1=0$

8. Find the equation of the straight line which is perpendicular to the line with intercepts $(1,0)$ and $(0,1)$ and intersects it at $(1/2,1/2)$.

   Ans: $y=x$

9. Find the equation of the straight line passing through $(1,1)$ and perpendicular to the straight line $x-2y+4=0$.

   Ans: $2x+y-3=0$

10. Where does the straight line with negative slope, passing through opposite corners of the rectangle with vertices $(1,1)$, $(1,2)$, $(4,2)$ and $(4,1)$, intersect the $y-$axis? Justify your answer by finding the equation of the line.

    Ans: $y = 2\frac{1}{3}$

11. Simplify $3^{-2.5}$ and rationalize the denominator of your answer.

Ans: $\sqrt{3}/27$.

12. Find all solutions to the quadratic equation $x^2 + x = 2x + 5$.

Ans: $\dfrac{1 \pm \sqrt{21}}{2}$.

13. Graph the function $f(x) = -x^2 + 4x$ noting the vertex, any symmetries and all intercepts.

14. Graph the function $f(x) = |x - 4| - 2$ noting any symmetries and all $x-$ and $y-$intercepts.

15. Graph the function $f(x) = \sqrt{(x-2)^2} + 2$ noting any symmetries and all $x-$ and $y-$intercepts.

16. Graph the function $f(x) = \dfrac{|x-1|}{x-1} + 3$.

17. Graph the function $f(x) = \begin{cases} x^2 + 2x + 3, & \text{for } x \geq -1 \\ x + 3, & \text{for } x < -1. \end{cases}$

18. Graph the function given by $f(x) = \begin{cases} \sqrt{x}, & \text{for } x \geq 0 \\ 1 - x, & \text{for } x < 0 \end{cases}$.

19. Graph the function given by $f(x) = \begin{cases} x, & \text{for } x \geq 1 \\ x^2, & \text{for } x < 1 \end{cases}$.

20. Graph the function given by $f(x) = \begin{cases} x^2 - 2x + 1, & \text{for } x \geq 1 \\ -(x-1)^2, & \text{for } x < 1. \end{cases}$

21. Graph the function obtained by translating the graph of the function $f(x) = x^3$ three units to the left and one unit up.

22. Write the equation of the function obtained by translating the graph of the function $f(x) = x^2$ two units to the right and one unit up.

Ans: $f(x-2) + 1 = x^2 - 4x + 5$

23. Find the equation of the function obtained by translating the graph of the function $f(x) = |x|$ two units to the left and one unit down.

Ans: $f(x+2) - 1 = |x+2| - 1$

24. If $f(x) = \dfrac{1}{x}$ and $g(x) = \dfrac{\sqrt{x+7}}{x+2}$:

(a) Find an expression for the composition function $(f \circ g)$.

Ans: $(f \circ g) = \dfrac{x+2}{\sqrt{x+7}}$.

(b) Describe the domain of $(f \circ g)$ using interval notation.

Ans: All x in the interval $(-7, \infty)$ except $x = -2$.

25. Find an expression for the composition $(f \circ g)(x)$, where $f(x) = \dfrac{1}{x}$ and $g(x) = \sqrt{x+1} + \sqrt{x}$. Describe the domain of $(f \circ g)(x)$, using interval notation.

Ans: $\dfrac{1}{\sqrt{x+1} + \sqrt{x}}$ Domain: $[0, \infty)$

26. Find both $\lim\limits_{x \to \pm\infty} \dfrac{1}{\sqrt{x}}$, if either or both exist(s), otherwise state why either or both fail(s) to exist.

Ans: $\lim\limits_{x \to \infty} \dfrac{1}{\sqrt{x}} = 0$ but $\lim\limits_{x \to -\infty} \dfrac{1}{\sqrt{x}}$ does not exist since the function $\dfrac{1}{\sqrt{x}}$ is undefined for negative x.

27. Find both $\lim\limits_{x \to 0^{\pm}} \dfrac{1}{\sqrt{x}}$, if either or both exist(s), otherwise state why either or both fail(s) to exist.

Ans: $\lim\limits_{x \to 0^+} \dfrac{1}{\sqrt{x}} = \infty$ but $\lim\limits_{x \to 0^-} \dfrac{1}{\sqrt{x}}$ does not exist since the function $\dfrac{1}{\sqrt{x}}$ is undefined for negative x.

28. Find both $\lim\limits_{x \to \pm\infty} \dfrac{1-x}{1+x}$.

Ans: -1.

29. Find $\lim\limits_{x \to 4} \dfrac{\sqrt{1+2x}}{x-1}$.

Ans: 1

30. Find $\lim\limits_{x \to 2} \dfrac{x^2 - x - 2}{2x - 4}$.

Ans: 3/2

31. Find $\lim_{x \to -1} \dfrac{x^2 + 2x + 1}{x + 1}$.

Ans: 0

32. Find $\lim\limits_{x \to -3} \dfrac{x^2 + 8x + 15}{x + 3}$.

Ans: 2

33. Find $\lim_{x \to 2^-} \sqrt{2-x}$, $\lim_{x \to 2^+} \sqrt{2-x}$ and $\lim_{x \to 2} \sqrt{2-x}$, whichever exist(s).

Ans: The first limit is zero and the last two do not exist.

34. Is the function $f(x) = \begin{cases} x^3 & \text{if } x \geq 1 \\ 1 - x & \text{if } x < 1 \end{cases}$ continuous at $x = 1$? Explain your answer in terms of limits.

Ans: Not continuous at 1 since $\lim\limits_{x \to 1^-} f(x) \neq \lim\limits_{x \to 1^+} f(x)$.

35. Is the function $f(x) = \begin{cases} x + 1 & \text{if } x \geq -1 \\ x & \text{if } x < -1 \end{cases}$ continuous at $x = -1$? Explain your answer in terms of limits.

Ans: Not continuous at -1 since $\lim_{x\to -1^-} f(x) \neq \lim_{x\to -1^+} f(x)$.

36. Is the function $f(x) = \begin{cases} \frac{2x}{|x|} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$ continuous at $x = 0$? Explain your answer in terms of limits.

Ans: No, since $\lim_{x\to 0} f(x)$ does not exist

37. Is the function $f(x) = \begin{cases} \frac{3(x^2-1)}{x-1} & \text{if } x \neq 1 \\ 6 & \text{if } x = 1 \end{cases}$ continuous at $x = 1$? Explain your answer in terms of limits.

Ans: Yes, $\lim_{x\to 1} f(x) = f(1)$

38. Is the function $f(x) = \begin{cases} 2 & \text{if } x > -1 \\ 1 & \text{if } x = -1 \\ x^2 + 1 & \text{if } x < -1 \end{cases}$ continuous at $x = -1$? Explain your answer in terms of limits.

Ans: No, $\lim_{x\to -1} f(x) \neq f(-1)$

39. Is the function $f(x) = \begin{cases} \sqrt{x+4} & \text{if } x \geq 0 \\ -x + 2 & \text{if } x < 0 \end{cases}$ continuous at $x = 0$? Explain your answer in terms of limits.

Ans: Yes, $\lim_{x\to 0} f(x) = f(0)$

40. Is the function $f(x) = \begin{cases} |x| & \text{if } x \geq -1 \\ 1 & \text{if } x < -1 \end{cases}$ continuous at $x = -1$? Explain your answer in terms of limits.

Ans: Yes, $\lim_{x\to -1} f(x) = f(-1)$

41. Is the function $f(x) = \begin{cases} x^2 & \text{if } x \geq 1 \\ 1 - x & \text{if } x < 1 \end{cases}$ continuous at $x = 1$? Explain your answer in terms of limits.

Ans: No, $\lim_{x\to 1} f(x)$ does not exist.

42. Graph the function $f(x) = \dfrac{(x+2)(x-1)^2}{x^3 - x}$, noting the domain of definition, all intercepts, horizontal and vertical asymptotes and points of discontinuity.

Ans: The function has three points of discontinuity located at $x = \pm 1$ and $x = 0$; two vertical asymptotes located at $x = 0$ and $x = -1$; one horizontal asymptote located at $y = 1$ and one x intercept located at $x = -2$.

43. Graph the function $f(x) = \begin{cases} \frac{(x+2)(x-1)^2}{x^3-x} & \text{if } x \neq 1 \\ 0 & \text{if } x = 1 \end{cases}$, noting the domain of definition, all intercepts, horizontal and vertical asymptotes and points of discontinuity.

Ans: This is the same as Problem 42 except this function is defined, continuous and has a zero at $x = 1$. The asymptotes are unchanged.

44. Find $\lim_{h\to 0} \dfrac{(2+h)^{-3} - 1/8}{h}$.

Ans: $-3/16$. This should be immediately recognized as $f'(2)$, where $f(x) = \dfrac{1}{x^3}$.

45. Write the equation of the line tangent to the graph of $f(x) = x^{-2}$ at the point $(2, 1/4)$.

Ans: $y = -\frac{1}{4}x + \frac{3}{4}$.

46. Is the function $f(x) = |x+4| - 1$ differentiable at either $x = 1$ or $x = -4$? Justify by considering limits. Compute $f'(1)$ and $f'(4)$ where possible.

Ans: $f(x)$ is not differentiable at $x = -4$, since

$$\lim_{h \to 0^-} \frac{f(-4+h) - f(-4)}{h} = -1 \neq 1 = \lim_{h \to 0^+} \frac{f(-4+h) - f(-4)}{h}.$$

However $f'(1) = 1$.

47. A body and travels according to the law $s(t) = 16t^2 - \frac{2}{3}t$, where $s(t)$ is the distance the body has travelled in feet immediately after t seconds. What is the velocity (rate of change of distance with respect to time) immediately after 3 seconds?

Ans: $95\frac{1}{3}$ ft./sec.

48. What is the average velocity immediately after 3 seconds of the body in the previous problem?

Ans: $47\frac{1}{3}$ ft./sec.

49. Find the profit function if the demand function is $D(x) = -.02x + 36$ and the supply function is $S(x) = .03x - 32$.

Ans: $P(x) = -.05x^2 + 68x$

50. Find the equilibrium quantity if the demand function is $D(x) = -.02x+32$, and the supply function is $S(x) = .03x-38$.

Ans: 1,400

51. Find the cost of producing the 10^{th} bicycle if the cost of producing x bicycles is $C(x) = 0.1x^2 + 12x + 60$.

Ans: 13.90

52. Find the fixed (or start up) cost and the profit function $P(x)$ if the cost function is $C(x) = 20x + 1250$ and the revenue funcion is $R(x) = 50x - .1x^2$.

Ans: 1,250 $P(x) = -0.1x^2 + 30x - 1250$.

53. Simplify $2^{-\frac{9}{2}}$. Use radical notation and rationalize the denominator of your answer.

a) $\frac{\sqrt{2}}{8}$

b) $\frac{\sqrt{2}}{16}$

c) $\frac{\sqrt{2}}{32}$

d) $\frac{\sqrt{2}}{64}$

e) $\frac{\sqrt{2}}{128}$

Ans: **c**

54. There is a linear relation between temperatures given in Celsius and Fahrenheit. At sea level water freezes at 0 degrees Celsius and 32 degrees Fahrenheit, whereas it boils at 100 degrees Celsius and 212 degrees Fahrenheit. What Fahrenheit temperature corresponds to -40 degrees Celsius?

a) -13 degrees

b) -32 degrees

c) -40 degrees

d) -55 degrees

e) -66 degrees

Ans: **c**

55. What is the $y-$intercept of the straight line with x intercept $(2,0)$ and which is perpendicular to the line $x-y=1$?

a) $(0,0)$

b) $(0,1/2)$

c) $(0,1)$

d) $(0,3/2)$

e) $(0,2)$

Ans: **e**

56. Find the $x-$coordinate of the vertex of the quadratic function $f(x)=26+4x-2x^2$.

a) 1

b) -1

c) 0

d) 2

e) -2

Ans: **a**

57. Suppose the revenue from the sale of x bags of pretzels is $R(x)=.75x$ dollars, and the cost of making x bags is $C(x)=.60x+100$ dollars. Find the fewest number of (full) bags the manufacturer must produce to make a profit.

a) 332

b) 387

c) 489

d) 543

e) 667

Ans: **e**

58. Find $\lim\limits_{x\to 2^-} \dfrac{(x-4)(x+3)}{x(x-2)}$.

a) 0

b) 1

c) ∞

d) $-\infty$

e) The limit does not exist.

Ans: **c**

59. Which of the following statements, a) through e), is/are true concerning the function

$$f(x) = \begin{cases} \dfrac{5x^2 - 10x - 15}{x^2 - x - 6} & \text{if } x \neq 3 \\ 4 & \text{if } x = 3 \end{cases} ?$$

a) $f(x)$ is continuous for all $x \neq -2$ and $x \neq 3$.

b) $\lim\limits_{x \to 4^-} f(x)$ does not exist.

c) $f(x)$ has two vertical asymptotes.

d) $f(x)$ has two horizontal asymptotes.

e) $\lim\limits_{x \to \infty} f(x) = \infty$.

Ans: **a**

60. Find $\lim\limits_{h \to 0} \dfrac{\dfrac{1}{(2+h)^2} - \dfrac{1}{2^2}}{h}$.

a) 0

b) $\dfrac{-1}{8}$

c) $\dfrac{-1}{4}$

d) $\dfrac{-1}{2}$

e) -1

Ans: **c**

61. Find the instantaneous rate of change of the function $f(x) = \dfrac{1}{x^4} + 6\sqrt[3]{x}$ at $x = 1$.

a) 0

b) 1

c) -1

d) 2

e) -2

Ans: **e**

62. A brick comes loose from the top of a 144 foot building. Its distance s (in feet) from the street at the time t (in seconds) is given by $s(t) = 144 - 16t^2$. What is the speed at which the brick is travelling when it hits the ground? Recall that the *speed* is the absolute value of the velocity.

a) 24 ft./sec.

b) 36 ft./sec.

c) 48 ft./sec.

d) 72 ft./sec.

e) 96 ft./sec.

Ans: **e**

63. What is the average speed of the brick in the above problem from the time it begins to fall until it hits the ground?

a) 12 ft./sec.

b) 18 ft./sec.

c) 24 ft./sec.

d) 36 ft./sec.

e) 48 ft./sec.

Ans: **e**