• **Differentiation of e^x.** Recall that the slope of the secant chord or line connecting the two points $(0, f(0))$ and $(h, f(h))$ on the graph of $f(x)$ is given by $m_{\sec} = \frac{f(h)-f(0)}{h}$. If $f(x) = e^x$, then $m_{\sec} = \frac{e^h-1}{h}$, so that taking the limit, we find

$$f'(0) = \lim_{h\to 0} \frac{e^h - 1}{h}.$$

You can estimate this limit to be 1 with a calculator. If the caclulator is a graphics calculator then you can superimpose the graph of the line with equation $y = x + 1$, on the graph of the exponential function in a small area about $(0, 1)$ to visually observe that the straight line is indeed tangent to the exponential at the point $(1, 0)$.

Note also that $lim_{h\to 0^+}(1 + h)^{1/h} = \lim_{n\to\infty}(1 + \frac{1}{n})^n = e$ implies

$$lim_{h\to 0^+} \frac{e^h - 1}{h} = lim_{h\to 0^+} \frac{\left((1 + h)^{1/h}\right)^h - 1}{h} = \frac{h}{h} = 1.$$

Using the fact that the derivative of the exponential is 1 at $x = 0$, we can then easily compute the derivative of the exponential at an arbitrary x from the definition. To wit: $D_x e^x = \lim\limits_{h\to 0} \dfrac{e^{x+h} - e^x}{h} = e^x \left(\lim\limits_{h\to 0} \dfrac{e^h - 1}{h}\right) = e^x.$

Thus $D_x e^x = e^x$ for all x. Hence, the function $f(x) = e^x$, doesn't change under differentiation.

• **Differentiation of $\ln x$.** Since e^x and $\ln x$ are inversely related functions (see notes on §5.2), we must have $e^{\ln x} = x$, for $x > 0$. Differentiating both sides of this equation, with the help of the chain rule, one finds $xD_x \ln x = 1$. Therefore

$$D_x \ln x = \frac{1}{x} \text{ for } x > 0.$$

• **Differentiation of b^x and $\log_b x$ for $0 < b \neq 1$.** Setting $b^x = e^{x \ln b}$, differentiation shows $D_x b^x = e^{x \ln b} \ln b$. Hence

$$D_x b^x = (\ln b)b^x.$$

Noting that b^x and $\log_b x$ are also functions which are inversely related, one must have $b^{\log_b x} = x$ for all $x > 0$. Once again, differentiation gives

$$D_x \log_b x = \frac{1}{(\ln b)x} \text{ for } x > 0.$$

Problem (change of basis formula). Suppose $0 < a \neq 1$ and $0 < b \neq 1$, show that

$$\log_b x = \log_b a \log_a x.$$

Problem. Use the above change of basis formula to rederive the formula for $D_x \log_b x$.