• **Absolute Extrema.** Let $f(x)$ be a function defined on a closed interval $I = [a, b]$. If there exists some c in the interval I such that $f(c) \geq f(x)$ for all x in the interval then the point $(c, f(c))$ on the graph of the function is called an *absolute maximum* of the function on I. If on the other hand, there exists some c in the interval I such that $f(c) \leq f(x)$ for all x in the interval then the point $(c, f(c))$ on the graph of the function is called an *absolute minimum* of the funcion on I. In other words absolute maxima are the highest points on the graph of the function over the interval I, whereas absolute minima are the lowest. Some functions have several absolute extrema whereas others have none. As an example of the latter, consider the two piece function defined by

$$f(x) = \begin{cases} \frac{1}{x}, & \text{for } x \neq 0 \\ 0, & \text{for } x = 0 \end{cases}.$$

Clearly, $f(x)$ has no absolute extrema on the interval $I = [-1, 1]$. Note also that this function is not continuous at every point of I.

• **Theorem.** Every function which is continuous at each point of a closed bounded interval I, has both an absolute maximum and an absolute minimum over I.

Recall that every function which is differentiable at a point is also continuous at that point. Thus we have the following Corollary.

• **Corollary.** Every function which is differentiable at each point of a closed bounded interval I, has both an absolute maximum and an absolute minimum over I. Moreover each absolute extrema must always occur at a point $(c, f(c))$ on the graph where either

i) c is a critical number of the interval or
ii) c is an end point of the interval.

• **Example.** Identify the absolute extrema of the function $f(x) = (x-1)^2$, over the interval $I = [1/2, 2]$.

The above Corollary implies that the absolute extrema must occur at one of the endpoints: 1/2 or 2, or at the critical number: $c = 1$. Since $f(1/2) = 1/4, f(1) = 0$ and $f(2) = 1$, we find $(1, 0)$ to be an absolute minimum and $(2, 1)$ to be an absolute maximum. If we change the interval I to be $[0, 2]$, then $(1, 0)$ would still be an absolute minimum whereas both $(0, 1)$ and $(2, 1)$ would be absolute maxima. This is then an example where the absolute maxima is not unique. Can you construct an example where the absolute minima is not unique?

Problem. Suppose that $f(x)$ is any third degree polynomial such that its derivative $f'(x)$ does not have two distinct real roots. Show that the absolute extrema of $f(x)$ occur at and only at the end points of an interval $[a, b]$. *Hint: Consider* $f(x) = x^3 + 3x^2 + 3x + 4$.
Problem. A farmer wishes to construct a pig-pen in the shape of a right triangle using a "long" barn, in lieu of fencing, as its hypotenuse. What is the largest area that he can enclose with 20 foot of fencing? Find the maximum area if he uses the barn as one of the legs of the triangle. Which of the two ways should he build the triangle to maximize the area?