• **Rules of Exponents.** If x, y are nonzero real numbers and m, n are nonzero integers then:

i) $x^0 = 1$, $xxx \cdots x$ (m times) $= x^m$ and $\frac{1}{x}\frac{1}{x}\frac{1}{x} \cdots \frac{1}{x}$ (m times) $= x^{-m}$

ii) $x^m x^n = x^{m+n}$, $(xy)^m = x^m y^m$ and $(x^m)^n = x^{mn}$

iii) $(\frac{x}{y})^m = \frac{x^m}{y^m}$ and $\frac{x^m}{x^n} = x^{m-n}$

iv) $x^{\frac{m}{n}} = (x^{\frac{1}{n}})^m = (x^m)^{\frac{1}{n}} = (\sqrt[n]{x})^m = \sqrt[n]{x^m} \quad (x > 0)$.

Problem. Simplify $64^{-\frac{5}{3}}$.

• **Quadratic Equations.** A quadratic equation is one of the form

$$ax^2 + bx + c = 0, \text{ where } a, b, \text{ and } c \text{ are constants with } a \neq 0.$$

Any (real) number r which satisfies this equality is called a (real) *root.* Its roots can be found by either factoring the left side of the equation, completing the square or using the quadratic formula as given by:

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The quantity $b^2 - 4ac$ is called the *radical* or *discriminent*; the quadratic equation has two, one, or no roots according as it's radical is positive, zero or negative respectively.

Problem. Find the roots of $2x^2 - 5x - 3 = 0$ both by factoring the equation and using the quadratic formula.

Problem. Determine how many (real) roots each of the following quadratic equations has by considering the radical/discriminent.

i) $x^2 + x + 1 = 0$,

ii) $4x^2 + 6x - 4 = 0$,

iii) $4x^2 + 16x + 16 = 0$.

Problem. Solve the quadratic equation $x^2 - 2x - 2 = 0$ by completing the square.

Problem. Derive the quadratic formula by completing the square of the general quadratic equation $ax^2 + bx + c = 0 \ (a \neq 0)$.

• **Functions.** A *function* is a rule that assigns to each element of one set a unique element of another set. The former set is called called the *domain* and the latter is called the *range* of the function. If we denote the function or rule by f and a typical but arbitrary element of the domain by x, then the element y in the range assigned to x by f is usually denoted by $f(x)$. Thus we write $y = f(x)$; x is called the *independent variable or argument of f* and y, which depends on x, is called the *dependent variable.* For example, the area A of a circle with radius r is dependent on the independent variable r. The function or rule f that describes this is known to be $f(r) = \pi r^2$. We write $A = f(r)$. The domain of f is then all radii possible or all positive real numbers and the range is all obtainable areas or again all positive real numbers.

Problem. Find the domain and range of the functions given by:

i) $f(x) = \sqrt{2x - 1}$ and

ii) $g(x) = \dfrac{x^2 - 1}{x^2 + 1}$.

A *zero* or *root* of a function f is any number x_0 such that $f(x_0) = 0$. For example the zeroes of $f(x) = ax^2 + bx + c$ are simply those x satisfying the equation $ax^2 + bx + c = 0$.

Problem. Find the zeroes of the two functions f and g defined in the previous problem.