• **Marginal Rate of a Function.** If $f(x)$ is a function which is differentiable at a point x, then the derivative $f'(x)$ is also called the *marginal rate of f(x)* at the point x.

• **Income Tax Example.** If your marginal income tax rate is 28%, this does not mean that you pay 28 cents on every dollar that you earn. It only means that you pay 28 cents on every dollar above a certain base.

• **Marginal Rate of Functions in Economics.** If $f(x)$ is a function in economics such as the cost function, $C(x)$, revenue function $R(x)$ or profit function $P(x) = R(x) - C(x)$, then when x is *large* it follows that $\Delta x = 1$ is *small.* Thus for *large* x, the marginal cost, marginal revenue or marginal profit approximates the cost, revenue or profit (respectively) of the $(x+1)^{th}$ item at production level x. That is,

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \text{ (approximately equal) } \frac{f(x+1) - f(x)}{1} = f(x+1) - f(x).$$

• **Maximum Profit.** It is frequently of interest to find the maximum profit. It can only occur, for a differentiable function $P(x)$, at those points x where the marginal profit, $P'(x)$, is 0. If for example, the profit function is a quadratic function of the form $P(x) = ax^2 + bx + c$ where $a < 0$ (which is frequently the case used in our text) then it follows that $P'(x) = 0$ when and only when $x = -\frac{b}{2a}$. Thus the maximum profit is $P\left(-\frac{b}{2a}\right)$. Recall that $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$ is the vertex of the graph of the quadratic.

Problem. The cost of producing x radios is $C(x) = 800 + 30x + 6x^2$ (dollars). Find the marginal cost at production level 500 and compare this with the cost of producing the 501^{st} radio.

• **Product Rule.** If $f(x)$ and $g(x)$ are two functions each of which is differentiable at x, then the derivative of their product at x is given by:

$$D_x\left(f(x)g(x)\right) = f'(x)g(x) + f(x)g'(x).$$

• **Triple Product Rule.** If $f(x), g(x)$ and $h(x)$ are three functions, each of which is differentiable at x, then the derivative of their product at x is given by:

$$D_x\left(f(x)g(x)h(x)\right) = f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x).$$

• **Quotient Rule.** If $f(x)$ and $g(x)$ are two functions each of which is differentiable at x and if $g(x) \neq 0$ then the derivative of their quotient at x is given by:

$$D_x\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{\left(g(x)\right)^2}.$$

Problems. i) Derive the triple product rule from the product rule.

ii) Generalize the triple product rule to a product of four functions and use the product and triple product rule to verify your generalization.

iii) Let $f(x)$ be a function which is differentiable but not 0 at x. Use the triple product rule, the quotient rule but not the (yet to be established) chain rule to show that

$$D_x\left(\frac{1}{\left(f(x)\right)^3}\right) = -3\frac{f'(x)}{\left(f(x)\right)^4}.$$

Hint: Apply the quotient rule to $\frac{1}{f(x)}$ *and then apply the triple product rule.*