• **Rules for Differentiation.** If $f(x)$ and $g(x)$ are functions which are differentiable at x and C is the constant function $F(x) = C$, then the following rules of differentiation are easily derived and should be memorized by the astute student.

i) Constant rules: If C is an arbitrary constant then

a) $D_x C = 0$, and

b) $D_x Cf(x) = CD_x f(x)$.

ii) Power rule: $D_x x^n = nx^{n-1}$ for all rational numbers n.

a) Setting $n = 0$, implies $D_x 1 = 0$.

b) Setting $n = 1/2$, shows $D_x\sqrt{x} = \dfrac{1}{2\sqrt{x}}$.

c) Setting $n = 1, 2, 3, \ldots$, shows $D_x x = 1, D_x x^2 = 2x, D_x x^3 = 3x^2, \ldots$

d) Setting $n = -1, -2, -3, \ldots$, shows $D_x \frac{1}{x} = -\frac{1}{x^2}, D_x \frac{1}{x^2} = -\frac{2}{x^3}, D_x \frac{1}{x^3} = -\frac{3}{x^4} \ldots$

iii) The sum and difference rule: The sum or difference of the derivatives is the derivative of the sum or difference. Specifically, $D_x f(x) \pm D_x g(x) = D_x (f(x) \pm g(x))$. Rule iii), of course can be extended to more than two functions. In particular it, along with i) and ii), applies to differentiating polynomials. For example, $D_x \left(x^3 + 4x^2 + 3x + 2\right) = 3x^2 + 8x + 3$.

Problems. i) Differentiate $f(x) = x^3 - \dfrac{1}{x^3} + \dfrac{1}{\sqrt{x}}$.

ii) Differentiate $f(x) = mx + b$ where m and b are constants.

iii) Differentiate $f(x) = \dfrac{x^2 + 1}{\sqrt{x}}$.

iv) Write the equation of the line tangent to the function $f(x) = 1/x$ at $(1, 1)$.

v) Write the equation of the line tangent to the function $f(x) = \sqrt{x}$ at $(4, 2)$.

• **Average and Instantaneous Rates of Change.** The *average rate of change* over an interval $[a, b], (a < b)$ of a function $f(x)$ is defined by $\dfrac{f(b) - f(a)}{b - a}$. Of course this is nothing more than m_{sec} as defined in the previous lecture. The *instantaneous rate of change at the point* a is defined by $\lim_{b\to a} \dfrac{f(b) - f(a)}{b - a}$ if this limit exists. Setting, $h = b - a$ one sees that the instantaneous rate of change at a is simply the derivative $f'(a)$.

• **Velocity.** If a function $s(t)$ measures the distance a body has traveled at time t, the average and instantaneous rates of change are called the *average and instantaneous velocity* respectively. In as much as the velocity $v(t)$ can be negative (eg. when an automobile is backing up), it is convenient to define the *speed* as the absolute value, $|v(t)|$, of the velocity.

Problems. The following problems were taken off a previous exam.

i) A brick comes loose from the top of a 144 foot building. If its distance s (in feet) from the street at the time t (in seconds) is given by $s = 144 - 16t^2$. At what speed is the brick is travelling when it hits the ground? This seed is called the *impact speed.*

ii) What is the average speed of the brick in the above problem from the time it begins to fall until it hits the ground?