

MATH 110: SUPPLEMENTARY REVIEW PROBLEMS FOR CHAPTER 5

1. Sketch the graphs of each of the following functions.

(a) $f(x) = 2^{x-1}$

Ans: Exponential growth, $f(x) = Ab^x$ where $A = \frac{1}{2}, b = 2$.

(b) $f(x) = \left(\frac{1}{2}\right)^{1-x}$

Ans: Exponential growth, $f(x) = Ab^x$ where $A = \frac{1}{2}, b = 2$ as above.

(c) $f(x) = (.4)^x + e$

Ans: Exponential decay translated up e units.

(d) $f(x) = ex^2 - 2ex + 1$

Ans: A parabola opening up with vertex at $(1, 1 - e)$; find the roots of this parabola as an exercise.

(e) $f(x) = \log_{1/2} x$

2. Find the horizontal asymptotes of each of the following functions.

(a) $f(x) = \frac{e^x + 1}{1 - e^x}$

Ans: $y = \pm 1$

(b) $f(x) = \frac{e^{2x} + e^x + 1}{2e^{2x} - e^x - 4}$

Ans: $y = 1/2$, and $y = -1/4$.

3. Find the domain of each of the following functions.

a) $f(x) = \ln \frac{1}{1-x^2}$

Ans: $|x| < 1$

b) $f(x) = \ln(x-1)^2$

Ans: $x \neq 1$

c) $f(x) = 2 \ln(x-1)$

Ans: $x > 1$, note the two functions defined in (b) and (c) only agree on $(1, \infty)$

4. Solve $\frac{(\ln x) + 1}{(\ln x) - 1} = 2$ for x .

Ans: $x = e^3$.

5. Sketch the graphs of each of the following functions without using calculus.

(a) $y = \ln e(x+1)$

Ans: The graph of $\ln x$ translated up and to the left by 1 unit.

(b) $y = \ln \frac{1}{x}$

Ans: Since $\ln \frac{1}{x} = -\ln x$, the graph is just that of the graph of $\ln x$ reflected about the x -axis.

(c) $y = \ln(ex)$

Ans: Since $\ln ex = \ln x + 1$ the graph is that of $\ln x$ translated up by one unit.

(d) $y = \ln(-x)$

Ans: The graph is that of $\ln x$ reflected about the y -axis

6. If 80g of radioactive radium decays to 70g in 200 years, what is the half life of radium?

Ans: $200 \frac{\ln 2}{\ln 8 - \ln 7}$

7. Determine the derivative of each of the following functions.

(a) $y = \sqrt{\frac{e^x + 1}{e^x - 1}}$

Ans: $-\frac{e^x}{\sqrt{(e^{2x} - 1)(e^x - 1)}}$

(b) $f(x) = (e^{-x} + 2)^{-4}$

Ans: $-4 \frac{e^{-x}}{(e^{-x} + 2)^5}$

8. In 7(a) and (b) determine the intervals over which the function is increasing and over which it is decreasing.

Ans: (a) Decreasing on $(0, \infty)$, not defined elsewhere.

Ans: (b) Increasing everywhere since the derivative is always positive.

9. For each of the functions below, determine all critical numbers **and** all relative extrema.

(a) $y = x^2 \cdot 2^x$

Ans: Cr Pts $x = 0, x = -\frac{2}{\ln 2}$; Rel Max $x = -\frac{2}{\ln 2}$; Rel Min $x = 0$

(b) $f(x) = (1 + x)1.5^x$

Ans: Cr Pt and Rel Min at $x = -\frac{\ln 1.5 + 1}{\ln 1.5}$

10. Use implicit differentiation to find $\frac{dy}{dx}$ for $x + y + x2^y = 0$.

Ans: $\frac{dy}{dx} = -\frac{1 + 2^y}{1 + x2^y \ln 2}$

11. Find the equation of the line tangent to the graph of each of the following at the indicated point.

(a) $y = \sqrt{\frac{\ln x + 1}{\ln x - 1}} \quad (x = e^2).$

Ans: $y = -\frac{1}{\sqrt{3}e^2}x + \frac{4}{\sqrt{3}}$

(b) $y = e^{\ln x} \quad (x = 1).$

Ans: $y = x$

(c) $y = \ln(x + 1)^{100} \quad (x = e - 1).$

Ans: $y = \frac{100}{e}(x + 1 - e) + 100$

12. For each of the functions below, determine all critical numbers and all relative extrema.

(a) $y = x \ln x$

Ans: Cr Pt and Rel Min at $x = 1/e$

(b) $y = \frac{\log_{10} x}{x}.$

Ans: Cr Pt and Rel Min at $x = e$

(c) $y = x^{1/x}$

Ans: Cr Pt and Rel Min at $x = \frac{1}{e}$

13. Let a, b be positive numbers, neither of which is 1. Show that $(\log_a b)(\log_b a) = 1.$

Ans: Set $z = \log_a b.$ Then $a^z = b.$ Taking \log_b of both sides of this equation implies the assertion.

14. Sketch the graph of $f(x) = x \ln x.$ You may assume that $\lim_{x \rightarrow 0^+} f(x) = 0.$

15. Given that the effective rate of a savings and loan is 7%, what is the nominal interest rate?

Ans: $\ln 1.07$

16. Suppose you invest \$5,000 at an interest rate of 5% per year. Use the rule of 70 to determine how long it will take for the value of your investment to become \$20,000.

Ans: 28 years

17. Ron and Doris want to establish an account that will be worth \$200,000 for their newly born daughter. If they invest \$25,000 at an annual interest rate of 10%, use either

the rule of 70 or the rule of 72, whichever is more appropriate, to estimate how old their daughter will be when they reach their goal.

Ans: 21.6 years