

MATH 110: SUPPLEMENTARY REVIEW PROBLEMS FOR CHAPTER 4

1. Find that number which most exceeds its square.

**Ans:**  $1/2$

2. Find two positive numbers whose sum is 20 and whose product is as large as possible.

**Ans:** 10, 10

For Problems 3 through 11, find all critical numbers and identify them as relative maxima or relative minima. Find all inflection points. Finally find and identify all absolute extrema on the given interval.

3.  $y = x^4 - 2x^2 + 2$  on  $[-1, 2]$ .

**Ans:** Crit. Nos.  $x = \pm 1, x = 0$ ; Rel. Max.  $(0, 2)$ ; Rel. Min  $(\pm 1, 1)$ ; Abs. Max.  $(2, 10)$ ; Abs. Min.  $(\pm 1, 1)$ . Infl. Pts.  $x = \pm \frac{1}{\sqrt{3}}$ .

4.  $y = \begin{cases} -x & x \leq 0 \\ 2x - x^2 & x > 0 \end{cases}$  on  $(-\infty, \infty)$ .

**Ans:** Crit. Nos.  $x = 0, x = 1$ ; Rel. Max.  $(1, 1)$ ; Rel. Min.  $(0, 0)$ ; No Abs. Extrema; No Infl. Pt.

5.  $y = x^4 - x^2$  on  $[-1, 1]$ .

**Ans:** Crit. Nos.  $x = 0, x = \pm \frac{1}{\sqrt{2}}$ ; Rel. and Abs. Max.  $(0, 0)$ ; Abs Max. also at  $(\pm 1, 0)$ ; Rel and Abs. Min.  $(\pm \frac{1}{\sqrt{2}}, -\frac{1}{4})$ ; Infl. Pt.  $\pm \frac{1}{\sqrt{6}}$ .

6.  $y = x - x^2$  on  $[0, 1]$ .

**Ans:** Crit. Nos.  $x = \frac{1}{2}$ ; Rel. and Abs. Max.  $(\frac{1}{2}, \frac{1}{4})$  Abs. Min.  $(0, 0), (1, 0)$ ; No Infl. pt.

7.  $y = x^2 - 4x + 3$  on  $[0, 3]$ .

**Ans:** Crit. Nos.  $x = 2$ ; Rel. and Abs. Min.  $(2, -1)$ .; Abs. Min.  $(0, 3)$ ; No infl. Pts.

8.  $y = \sqrt{4 - x^2}$  on  $[-2, 2]$ .

**Ans:** Crit. Nos.  $x = 0, x = \pm 2$ ; Rel. Max and Abs. Max.  $(0, 2)$ , Abs. Min.  $(\pm 2, 0)$ ; No Infl. Pts.

9.  $y = x^4 - 8x^3 - 270x^2$  on  $(-\infty, \infty)$ .

**Ans:** Crit. Nos.  $x = -9, x = 15$ ; , Rel. Min. at  $x = -9$  and  $x = 15$ , Rel. Max. at  $x = 0$ ; Abs. Min. at  $x = 15$ , Infl. Pts.  $x = -5$  and  $x = 9$ .

10.  $y = x^4 - \frac{x^3}{3} - 2x^2 + x - 1$  on  $(-\infty, \infty)$ .

**Ans:** Crit. Nos.  $x = \frac{1}{4}, x = \pm 1$ ; Rel. Min.  $(1, -\frac{4}{3})$ , and  $(-1, -\frac{8}{3})$ ; Abs. Min  $(-1, -\frac{8}{3})$ ; Abs. Max. at  $x = -\frac{1}{4}$ ; Infl. Pts. at  $x = -\frac{1}{2}, \frac{2}{3}$ ,

11.  $y = (x - x^2)^{-1}$  on  $[0, 1]$ .

**Ans:** Crit. Nos.  $x = \frac{1}{2}$ ; Rel. and Abs. Min  $(\frac{1}{2}, -4)$ ; No Infl. Pts.

Graph each of the functions in Problems 12 through 20.

12.  $f(x) = \frac{x^2}{x^2 - 1}$ .

**Ans:** Hor. Asympt.  $y = 1$ ; Vert. Asympt.  $x = \pm 1$ ; Rel. Max.  $(0, 0)$ ; Sym w/r  $y$ -axis.

13.  $f(x) = x - \frac{1}{x}$ .

**Ans:** Incr. on  $(-\infty, 0), (0, \infty)$ ; Vert. Asympt.  $x = 0$ ; Asympt.  $y = x$ . Sym. w/r  $(0, 0)$ .

14.  $f(x) = x + \frac{1}{x}$ .

**Ans:** Axis of Sym. and Asympt.  $y = x$ ; Sym. w/r  $(0, 0)$ ; Vert. Asympt.  $x = 0$ ; Rel. Max.  $(-1, -2)$ ; Rel. Min.  $(1, 2)$ .

15.  $f(x) = \frac{1}{x^2 + 4x + 3}$ .

**Ans:** Axis of Sym.  $x = -2$ ; Vert. Asympt.  $x = -2 \pm 1$ ; Hor. Asympt.  $y = 0$ ; Rel. Max.  $(-2, -1)$ .

16.  $y = |x^2 - 1|$ .

**Ans:** Sym. w/r  $y$ -axis; Rel. Max.  $(0, 1)$ ; Abs. Min. and Cusp.  $(\pm 1, 0)$ ; non negative everywhere.

17.  $f(x) = \sqrt{|x - 3|}$ .

**Ans:** Sym. w/r  $x = 3$ ; Cusp and Abs. Min. at  $(3, 0)$ ; nonnegative everywhere.

18.  $y = x^3 - 4x$ .

**Ans:** Sym. w/r  $(0, 0)$ ; Rel. Max. at  $x = -\frac{2}{\sqrt{3}}$ ; Rel. Min. at  $x = \frac{2}{\sqrt{3}}$ .

19.  $y = x^4 - 5x^2 + 4$ .

**Ans:** Sym. w/r  $y$ -axis; Abs. Min.  $x = \pm\frac{\sqrt{10}}{2}$ ; Rel. Max.  $(0, 4)$ . The graph looks like a  $W$ .

20.  $y = x + \frac{1}{x^2}$ .

**Ans:**  $y = x, x = 0$  are Asympt.; Rel. Min. at  $x = \sqrt[3]{2}$ ; Increasing for  $x < 0$  and  $x > \sqrt[3]{2}$ ; Decreases for  $0 < x < \sqrt[3]{2}$ .