REU 2014 COURSE DESCRIPTION - Lie Groups and Lie Algebras

Our main goal is to discuss the basic structure of Lie groups and Lie algebras.

We will assume as known introductory topics in analysis and topology: topological spaces, Euclidean topology, continuous functions, compact spaces, metric spaces, completeness, Heine-Borel theorem, Baire category. For standard references, see 1) and 2) below. If needed, some of these topics will be covered during the weekly seminar, as well as in the individual meetings with the groups.

We start with an introduction to differentiable and analytic manifolds. A Lie group is at the same time a group and a differentiable manifold, so some knowledge of differentiable manifolds is required. In particular we will describe here the construction of global solutions for involutive systems of differentiable equations on a manifold. We continue with general results about Lie groups and Lie algebras, the correspondence between Lie groups and Lie algebras, the exponential map, the Campbell-Hausdorff formula, and the fundamental theorems of Lie. The emphasize will be on examples. The last part of the course is devoted entirely to Lie algebras: the theorems of Lie and Engel about nilpotent and solvable Lie algebras; criterion for semisimplicity, namely that a Lie algebra is semisimple if and only if its Cartan-Killing form is nonsingular; Levi’s semisimple decomposition of a general Lie algebra into its radical and a semisimple factor. If time permits, we will try to learn about maximal semigroups with nonempty interior in nilpotent and solvable Lie groups. Standard references for general theory about Lie groups and Lie algebras are 4) and 5). The theory of maximal semigroups in nilpotent and solvable Lie groups is developed in 6).

**Bibliography**

2) W. Rudin, Principles of real analysis, Elsevier, 1998
3) W. Rudin, Real and complex analysis, McGraw-Hill, New York, 1966
4) Varadarajan, Lie groups, Lie algebras and their representations, Springer, 1984
5) M. Postnikov, Lie groups and Lie algebras, 1986