Bringing Eastern European Mathematical Traditions to North American Students

Paul D. Humke, Yulij Ilyashenko, and Serge Tabachnikov

Three Programs for Undergraduate Mathematics Majors Focus on Discovery and Exploration

This article presents descriptions of three noteworthy programs designed to expose undergraduate mathematics majors to a wider variety of mathematical experiences than is typically found in college courses. The programs share a common philosophical core, in that they all draw on the mathematical and intellectual traditions of Eastern Europe.

The courses offered in the programs are quite different from the usual undergraduate fare, as the emphasis is on discovery and depth of understanding rather than on "covering the material" and preparing for examinations. These are not summer programs; rather, they offer semester-long courses that run during the regular academic year, and credits can be transferred to the students' home institutions. By encouraging independent thinking and exploration, all three programs give students a taste of what it is like to do research in mathematics.

Further information about the programs is available on their respective websites. A list of such programs, together with other information of interest to undergraduate mathematics majors, may be found on the AMS website at `http://www.ams.org/employment/undergrad.html`.

—Allyn Jackson

Budapest Semesters in Mathematics

`http://www.stolaf.edu/depts/math/budapest/`

Paul D. Humke

Initiated by Paul Erdős, László Lovász, and Vera T. Sós in 1984, the Budapest Semesters in Mathematics (BSM) program offers a unique opportunity to North American undergraduates for a semester or a year of study in one of the world's hubs of mathematical activity. A wide variety of courses in all areas of mathematics are offered under the tutelage of eminent Hungarian teacher-scholars, most of whom have had years of teaching experience in North America. Classes are small and taught in English, and credits are transferable to the student's home institution. In keeping with the Hungarian tradition, professors closely monitor each individual student's progress. Considerable time is devoted to *problem solving* and encouraging *student creativity*. Emphasis is on depth of understanding rather than on the quantity of material. The imprint of the Hungarian tradition is particularly prominent in the lively regular BSM colloquium talks and in some particular courses.

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The courses Combinatorics and Topics in Graph Theory concentrate on combinatorial structures and algorithms, a stronghold of Hungarian mathematics. These courses, along with Theory of Computing, are an essential introduction to theoretical computer science. In Number Theory, especially the advanced course, one sees the hand of Paul Erdős, who had a profound influence on the subject.

The flagship course, Conjecture and Proof, was designed by Miklos Laczkovich to introduce students to the intrinsic excitement of mathematical discovery. Concepts, methods, ideas, and paradoxes that have startled or puzzled mathematicians for centuries are rediscovered and examined under the guidance of enthusiastic and experienced instructors. Topics covered range from ancient problems of geometry and arithmetic to modern discoveries in measure theory and mathematical logic; the emphasis, however, is on beauty and power rather than on subject matter.

Two semesters are offered each year: each semester is comprised of fourteen weeks of teaching and one week of comprehensive examinations. The fall term begins the first week of September and ends in mid-December, while the spring term begins the first week of February and ends in May. An intensive Hungarian language course begins about two weeks prior to the beginning of each semester. Although this course is optional, students who attend emerge from the noncredited eighty hours with solid *survival* Hungarian.

A typical course load is three to four mathematics courses and one or two intercultural courses each semester. The BSM program offers several levels of Hungarian Language and a variety of nonmathematics courses each term. Recent titles include The Making of Modern Central Europe, Hungarian Art and Culture, Historical Aspects of Mathematics, and Old World and New World Political Philosophy. Additional nonmathematics courses are available to BSM students through other American programs taught at College International.

Classes are held near the center of Pest at the College International campus in a tastefully renovated academic building complete with appropriately fast Internet connection, modern networking, fully accessible facilities, and an outdoor basketball court. Student accommodations, either in apartments with other BSM students or in private residences, are excellent, and living costs are modest.

During the past decade, participation in the BSM program has grown to about fifty students each semester, and to date BSM has over 1,200 alumna/alumni from more than two hundred colleges and universities spanning the spectrum of higher education in North America. The mean home school grade-point average of a participant is just over 3.7 on a scale of 4, with the mathematics grade-point average slightly higher. Other than the fact that participants are talented and motivated in mathematics, they are the most heterogeneous group I have ever been associated with. In general, BSM students are an adventurous lot who engage in their chosen activities with spirited determination and overwhelming energy. Several recent participants have used their "spare time" to play in one of the Liszt Academy's orchestras; others have served Hungarian relief organizations aiding refugees from neighboring countries; several have joined sports teams in swimming, soccer, and fencing—all have taken the opportunity to investigate various regions of Hungary and the surrounding European countries. Most students attend their first opera in Budapest, and many become devotees of the music, art, and general intellectual culture that is still vibrant (and affordable!) in this exotic, ancient city.

If a BSM student learns nothing else from the program, he or she learns "that mathematics is not a spectator sport". As Amy Myers (BSM spring 1993, now a professor at St. Joseph's University) wrote in September of 1994:

> I spent every Sunday evening with my study group sorting conjectures we had made individually during the week and discussing possible proofs. Living and studying with gifted individuals who share my love of mathematics was one of the most exciting and satisfying aspects of my Budapest experience.

Late last century Donald Knuth remarked, "The Hungarian educational system has been the most successful in pure mathematics." It is the practiced ability of these Hungarian teacher-scholars to put students eyeball to eyeball with the creative, mind-opening aspects of our discipline that lights fires in them. Recently, Andrew Gillette (BSM fall 2002, now a senior at Amherst College) wrote:

> This was exactly what I wanted out of study abroad—fantastic mathematics, a great community of students, and an incredible place to live. I know I'll never get another opportunity like I had last fall, and I'm really glad I took advantage of it. It's already changed me into a different person…I remember thinking as I was in Budapest that this was truly one of the coolest things I had done yet in my life and now, after returning, I'm convinced it was *the* coolest thing I've ever done.

It is the goal of BSM to imbed North American students into the mathematics community found in Budapest and to let them experience for themselves the excitement of being part of that community. As North American director my only regret is that the program did not exist when I was a student.

Math in Moscow

http://www.mccme.ru/mathinmoscow/

Yulij Ilyashenko

In 2001 the Independent University of Moscow (IUM) opened a one-semester program in mathematics for undergraduate and graduate students.

The IUM is a small, elite college for future research mathematicians. It was founded in 1991 by a group of renowned Russian mathematicians who now comprise the academic council of the university (headed by V. I. Arnold). The main goals of the IUM are to maintain the best traditions of the Moscow Mathematical School, to reverse the brain drain from Russia, and to increase relations between the international and the Russian mathematical communities. One of the main Russian mathematical traditions is to involve gifted students in research activity at very early stages of their studies. Another is to develop a creative approach to studying mathematics from the very outset. Not memorization of theorems and proofs, but the discovery of mathematics by the students themselves under the guidance of experienced teachers—this is a basic principle of Russian mathematical education. Even in the treatment of the most traditional subjects, the IUM student will find significant connections with contemporary research topics. The IUM curriculum is very flexible and easily follows new trends in mathematics.

The IUM is very small: no more than two hundred people are involved, including both faculty and students, but it is one of the centers of Moscow mathematical life. Leading Moscow mathematicians like Anosov, Arnold, Belavin, Feigin, Guseĭn-Zade, Khovanski, Kirillov, Novikov, Rudakov, Tikhomirov, Tsfasman, Vassiliev, and others give or have given lecture courses at the IUM. Several dozen such lecture courses have been published as preprints, and some of them have appeared as textbooks. The IUM, together with its partner, the Moscow Center of Continuous Mathematical Education, runs a small publishing house.

The IUM is situated in a newly reconstructed building in the historic center of Moscow. This building includes a library, a computer classroom, a cafeteria, and a book shop specializing in mathematics books. Computer facilities include free access to the Internet and electronic mail service.

The IUM runs the Math in Moscow (MIM) program, a one-semester study-abroad program for undergraduate and graduate students from the U.S. and Canada, and from other countries as well. The program offers mathematical and nonmathematical courses of different levels. The elementary courses are intended for well-motivated students with modest mathematical background; intermediate courses are of a level similar to honors courses for the senior students at Ivy League universities; advanced courses are like first-year graduate courses in the U.S. and Canada. Some examples of courses are: Geometric Foundations of Calculus, Topology I and II, Combinatorics, Non-Euclidean Geometry, Ordinary Differential Equations, Dynamical Systems, Partial Differential Equations, Introduction to Algebraic Geometry, Basic Representation Theory, Programming: From an Art to a Science, Russian History, Russian Literature, and Russian Language. There are many other courses, and the program varies from year to year.

In their contents the courses are organized according to the Russian tradition: the students are involved in the rediscovery of the subject. The form is similar to that of U.S. courses: there are two 90-minute lectures per week and weekly assignments. Part of the lecture time is devoted to the discussion of these assignments. Some courses at the MIM program have been given by Guseĭn-Zade, Kazarian, Tsfasman, Vassiliev, and other renowned mathematicians. The program has attracted students from Berkeley, Cornell, Harvard, MIT, University of Toronto, Université de Montréal, Yale University, and other U.S. and Canadian universities. At the end of the program many of the students note that they have never had such an extensive mathematical semester before.

The humanities courses offered by MIM help the students to plunge into Russian culture. The location of the IUM—twenty minutes' walking distance from the Kremlin and thirty minutes from the Bolshoy—also helps in that respect. During the semester, students can visit other cultural centers of Russia, including St. Petersburg.

To summarize, the main features of the MIM program are a modern, research-oriented curriculum; highly qualified professors with teaching experience in English; a comfortable building in the historical center of Moscow, with all the necessary facilities; Russian language and culture-learning options; modest tuition and low living expenses. The AMS and the Canadian Mathematical Society, together with the U.S. National Science Foundation and the National Science and Engineering Research Council of Canada, provide a certain number of fellowships to participants in the MIM program.

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Mathematics Advanced Study Semesters (MASS)

http://www.math.psu.edu/mass/

Serge Tabachnikov

MASS (Mathematics Advanced Study Semesters) is an intensive program for undergraduate students who are recruited every year from around the U.S. and brought to the main campus of Pennsylvania State University for one semester. The program was founded in 1996 by a group of Penn State faculty including G. Andrews, A. Katok, and S. Katok, and by A. Kouchnirenko of Moscow State University, who served as a director of the MASS program for the first two years.

MASS is unique among mathematics programs for undergraduates in this country. MASS students are literally immersed into mathematical studies: the full-time participants take no other classes. All the academic activities for a semester are specially designed and coordinated to reinforce each other and to introduce the students to research in mathematics. A key feature of the MASS experience is an intense and productive interaction among the students. By the end of a MASS semester one has a quantum leap effect: the achievements of the participants and their enthusiasm toward mathematics have increased much more dramatically than if they had been exposed to a similar amount of material over a longer time in a more conventional environment.

Students take three core courses from the general areas of analysis, algebra/number theory, and geometry/topology. Each course features three one-hour lectures per week, a weekly meeting conducted by a MASS teaching assistant, weekly homework assignments, a written midterm examination, and an oral final examination/presentation. Students conduct three individual research projects ranging from theoretical mathematics to computer implementation. Most of the projects are related to the core courses, while some are developed independently according to the interests and abilities of the student. There is a weekly two-hour interdisciplinary seminar run by the director of the MASS program (this author) which helps to unify all other activities. There is also the MASS colloquium, a weekly lecture series by distinguished mathematicians, visitors, or Penn State research faculty.

The elements of MASS (three courses, the seminar, and the colloquium) total 16 credit hours, transferable to MASS participants' home universities. The core courses are custom designed for the program and are available only to its participants. Each course addresses a fundamental topic not likely to be covered in the usual undergraduate (and, in many cases, even graduate) curriculum. For example, here is a list of MASS 2003 courses: Number Theory with Applications to Communication Networks (by W. Li), Topological Dynamics (by B. Kra), and Geometry and Relativity: An Introduction (by J. Roe).

The final examinations (three in total) have a format that is quite unusual for a U.S. university and that represents a creative development of European, in particular Russian, traditions. A student draws a random "ticket" that typically contains a theoretical question and a problem from the course. Then the student has an hour to prepare his or her answers in a "closed book" environment. The examination committee usually includes the course instructor, the teaching assistant, and another one or two Penn State faculty members. The answers to the ticket questions take only about a third of the oral examination. Another third is a presentation of the research project associated with the course; this presentation is prepared in advance and may involve slides, computer, etc. The last third of the exam is an open-format discussion with the committee.

A MASS colloquium is somewhat similar to the usual colloquium at a department of mathematics, but there are a number of important differences. A MASS colloquium talk should be accessible to undergraduates from beginning to end (and not just the first 5–10 minutes, as is often the case with its "grown-up" counterpart). A MASS colloquium is not just a survey of a certain area; it should contain a piece of concrete, hands-on mathematics (a proof of a theorem, a counterexample, a construction, etc.). Unlike most of the usual colloquia, the speaker does not necessarily focus on his or her own research: some of the talks are devoted to classical gems, "proofs from the Book". The range of speakers is broad too, from celebrated names (such as J. Conway or R. Penrose) to mathematicians in the early stages of their career; the only things in common are expository skill and contagious love of mathematics.

The required background of MASS students includes a full calculus sequence, basic linear algebra, and the first mathematical course on proofs (typically, discrete mathematics). Recruitment of participants is nationwide. They are selected based on their academic records, recommendation letters from faculty, and an essay. The number of MASS participants is about fifteen per semester. About 25 percent of MASS students come from small, mostly liberal arts schools, and about 40 percent from large state universities. The rest are Penn State students, mostly members of the Schreyer

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Honors College. A very small part come from elite schools, such as MIT or Caltech. A number of MASS students continue their focused mathematical education at the Math in Moscow program.

The majority of MASS graduates go on to graduate programs in mathematics. The distribution of the graduate schools is very wide. Without providing comprehensive data, here are a few: Cornell, Harvard, Indiana University, MIT, Princeton, Berkeley, Chicago, Georgia, Michigan, Texas, Wisconsin, Yale, and of course, Penn State. We already have a first MASS Ph.D.: a 1996 alumna, Suzanne (Lynch) Hruska, received a doctoral degree in mathematics from Cornell in 2002.

The MASS program makes a deep mark on all involved: the students (no matter whether they choose to specialize in mathematics or another field); the teaching assistants (handpicked from Penn State graduate mathematics students, in many cases MASS alumni themselves); and the instructors, who uniformly find teaching in MASS a challenging but very rewarding experience.