MASS PROGRAM AT PENN STATE

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The MASS program—Mathematics Advanced Study Semesters—is an intensive program for undergraduate students recruited every year from around the USA and brought to the Penn State campus for one semester. MASS belongs to a rare breed; we know of two other, somewhat similar, mathematical programs for American undergraduates, both based abroad: Budapest Semesters in Mathematics and Mathematics in Moscow; the former is in its "teens" (started in 1985) while the latter is just one year old. Mathematics Advanced Study Semesters at Penn State has turned six, and this seems to be a good time to reflect on the MASS community.

How it started

All three founders of the MASS program (the first two authors of this article and the first MASS director, A. Kouchnirenko) are steeped in the Russian tradition where interested students are exposed to a variety of mathematical endeavors, often of nonstandard kind, at an early age. By their senior undergraduate years such students are already budding professionals. We briefly describe this tradition in Appendix 1. The U.S. educational system is built on completely different principles, and interested young students are routinely encouraged to go fast through the required curriculum. Here a typical mathematically gifted high school student takes courses in local university and often is considered a nerd by his peers. The founders felt that there was a way to combine some of the best features of both traditions within the U.S. academic environment, namely, to gather a group of mathematics majors and to expose them to a substantial amount of interesting and challenging mathematics from the core fields of algebra, geometry and analysis which goes way beyond the usual curriculum.

The second author's first exposure to an intensive program for U.S. undergraduates was at the Mills College Summer Mathematics Institute for mathematically gifted undergraduate women. But why should such a program be all female? Why not to organize a co-educational program along the same lines whose participants would contribute a

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variety of experiences and backgrounds? The result was the SURI (Summer Undergraduate Research Initiative) program at Penn State in the summer of 1993 where all three future founders of the MASS program came together. During this program it became clear that a semester-long format would be even more productive for an intensive program organized mostly around advanced learning with elements of research initiation.

The following items were crucial to get MASS program off the ground:

- Cost for the participants should not increase compared to their home universities.
- The program should have its own room with several work stations in the Mathematics building.
- A special position and office should be secured for the MASS director.

It took three years to get the original financial commitment from the Penn State administration at various levels and to solve numerous logistic problems before the MASS program could begin.

Program description

The main components of MASS are:

- Three **core courses** on topics chosen from the areas of Analysis, Algebra/Number Theory, and Geometry/Topology. Each course features three, 1 hour lectures per week, a weekly meeting conducted by a MASS Teaching Assistant, weekly homework assignments, a written midterm exam, and an oral final examination/presentation.
- Individual student **research projects** ranging from theoretical mathematics to computer implementation. Most of the projects are related to the core courses while some are developed independently according to the interests and abilities of the student.
- A weekly 2 hour **interdisciplinary seminar** run by the director of the MASS program (the third author of this article), which helps to unify all other activities.
- The **MASS colloquium**, a weekly lecture series by distinguished mathematicians, visitors or Penn State research faculty.

All elements of MASS (3 courses, the seminar and the colloquium) total 16 credit hours, all listed as Honors classes, that are transferable to MASS participants' home universities. Additional recognition is provided through prizes for best projects and merit fellowships. Each student is issued a Supplement to the MASS Certificate which includes the list of MASS courses with credits, grades, final presentations, and special achievements. It also includes the descriptions of MASS courses, the list of MASS Colloquia and the description of MASS program exams. These supplements are very useful for the student's home institution equivalences and enhance the student's applications to graduate schools.

The main idea of the MASS Program, and its principal difference from various honors programs, math clubs, and summer educational or research programs, etc., is its comprehensive character. MASS participants are literally immersed into mathematical studies (since the program is very intense, its full-time participants are not supposed to take other classes). All the academic activities for a semester are specially designed and coordinated to enhance learning and introduce the students to research in mathematics. This produces a quantum leap effect: the achievement and enthusiasm of MASS students increases much more dramatically than if they had been exposed to a similar amount of material over a longer time in a more conventional environment.

A key feature of the MASS experience is an intense and productive interaction among the students. The environment is designed to encourage such interaction: a classroom is in full possession of MASS (quite a non-trivial arrangement in a large school such as Penn State!) and furnished to serve as a lounge and a computer lab outside of class times. Each student has a key and can enter the room 24 hours a day. The students live together in a contiguous block of dormitory rooms and they pursue various social activities together. The effect is dramatic: the students find themselves members of a cohesive group of like–minded people sharing a special formative experience. They quickly bond, and often remain friends after the program is over. They study together, attack problems together, debug programs together, collaborate on research projects and, most importantly, talk mathematics most of the time. Of course, this is exactly how "mature" mathematicians operate in their professional life! A necessary condition for this environment is the gathering of a "critical mass" of dedicated and talented students, which is one of the chief accomplishments of MASS.

Let us describe the MASS components in more detail. The core courses are custom designed for the program and are available only to its participants. We list the core courses offered since 1996 in Appendix 2. A brief look at this list reveals the main feature: each course addresses a fundamental topic which is not likely to be covered in the usual undergraduate (and, in many cases, even graduate) curriculum. Designing and teaching such a course, an instructor is challenged to reach a delicate balance between covering the basics, with which the students might be unfamiliar, and introducing an advanced material typically taught in topics courses.

Consider, for example, a MASS 2000 course "Finite groups, symmetry and elements of group representations" by A. Ocneanu. This class started with very fundamental facts about finite groups and their representations and proceeded to what is often referred to as "quantum topology": invariants of knots and 3-dimensional manifolds associated with statistical physics and Yang-Baxter equation. The course was received by the students with great enthusiasm and is likely to direct some of them toward this active area of research.

The final exams (three, in total) have a unique format. It is quite unusual for a US university and represents a creative development of a European tradition. A student draws a random "ticket" that typically contains a theoretical question from the course and a problem. Then the student has an hour to prepare her answers, and she has no access to literature or lecture notes during this hour. The answers to the ticket questions constitute only about a third of the oral examination. Another third is a presentation of the research project associated with the course; this presentation is prepared in advance and may involve slides, computer, etc. The last third of the exam is a discussion with the committee of three (including the course instructor, the teaching assistant and another Penn State faculty).

A MASS colloquium is similar to a usual colloquium at a department of mathematics with an important difference: a speaker cannot assume much of background material. Although this makes the speaker's task harder, we find that the quality of talks usually benefits from this restriction. To quote the opening sentences of an inspiring article by J. McCarthy "How to give a good colloquium" (see at www.math.psu.edu/colloquium/goodcoll.pdf): "Most colloquia are bad. They are too technical and aimed at too specialized an audience." This is precisely a sin that MASS colloquium is free of. As a result, along with MASS students, it is well attended by graduate students and faculty at the Department.

To preserve the intellectual effort that goes into MASS colloquium talks, a group of 2 or 3 MASS students is assigned to take notes and prepare a readable exposition of the talk. We also experiment with videotaping the talks.

Choosing the speakers, we always invite mathematicians known for their expository skills. We also try to represent as broad a spectrum of mathematical research as possible. We find it beneficial to combine very well known mathematicians with the ones in the early stage of their careers. A complete list of MASS colloquium talks is given in Appendix 3.

The MASS seminar plays many roles in the program. One of them is to introduce the students to the topics that, otherwise, are likely to "fall between cracks in the floor". For example, one of the seminar topics in 2001 was the classical configuration theorems of projective geometry: Pappus, Desargues, Pascal, Brianchon and Poncelet. Once projective geometry used to be a core subject in the university curriculum but nowadays it is perfectly possible to obtain a doctoral degree in mathematics without a single encounter with these facts. Another example: the theory of evolutes and involutes was a crowning achievement of Calculus to be included into textbooks. Alas, a contemporary student is not likely to see these things anymore. MASS seminar is a natural place to learn such a topic.

Another purpose of the seminar is to prepare the students for the upcoming MASS colloquium talks. A colloquium speaker is asked whether a certain material should be covered in advance so that the students get the most from the talk. For example, as a preparation for A. Kirillov's talk on Family Algebras in 2001, a whole MASS seminar was devoted to the basics of Lie groups and Lie algebras. Still another function of the seminar is to rehearse the students' presentations of the research projects on the final exam. This usually occupies the last quarter of the semester. Probably even more important function of the seminar is to bring about elements of unity of modern mathematics. Often identical or similar notions appear in different courses in various guises, and the seminar is the place to explore, develop and clarify these connections.

The summer program: REU and MASS Fest

The Penn State Summer REU (Research Experience for Undergraduates) program started in 1999 as an extension of MASS. Unlike MASS, this program is not unique: currently, there are about 40 REU programs in mathematics, available to undergraduate students in the USA. The Penn State REU is closely related to MASS: about half of its participants stay for the MASS semester in the fall. This makes it possible to offer research projects that require more than 7 weeks (the length of REU program) for completion.

Mathematical research usually includes three components: study of the subject, solving of a problem, and presentation of the result. These three components are present in the REU program: in addition to the traditional individual/small group research projects supervised by faculty mentors, the program includes two short courses, a weekly seminar, and the MASS Fest. See Appendix 4 for the list of REU courses.

MASS Fest is a 3-day conference at the end of the REU period at which the participants present their research. This is also a MASS alumni reunion. Along with the REU students, a number of guest speakers, mostly Penn State faculty, give expository talks at the conference.

Here are two examples of REU students' research projects.

"Simplices with only one integer point" (2 students; faculty mentor A. Borisov). The students found an effective procedure that allowed to describe all classes of simplices with vertices that have only integer coordinates and only one point with integer coordinates inside. Using computers they found all classes in dimensions 3 and 4.

"New congruences for the partition function" (1 student; faculty mentor K. Ono). This project started before the REU program began. Using the theory of Hecke operators for modular forms of half-integral weight, the student found an algorithm for primes $13 \le m \le 31$ which reveals $70,266$ new congruences of the form $p(An+B) \equiv 0 \pmod{m}$, where $p(n)$ denote the number of unrestricted partitions of a non-negative integer n. As an example, she proved that $p(3828498973n + 1217716) = 0 \pmod{13}$ for every integer n. The first three congruences were found in 1919 by Ramanujan, and after that finding new ones was considered a very difficult problem. The paper of this student has been accepted for publication.

We would like to emphasize a unique role played by the REU coordinator, M. Guysinsky, who has been coming to Penn State for the summer since 1999 as a visiting Assistant Professor supported by the VIGRE funds. He organizes all the RUE activities, including MASS fest, runs the seminar and supervises research projects, both suggested by other faculty, not present during the REU period, and offered by him. This requires an unusual combination of mathematicaal and pedagogical talents, and we are very fortunate to have found this combination in Guysinsky.

Participants

MASS participants are selected from applicants currently enrolled in US colleges or universities who are juniors, seniors or sometimes sophomores. They are expected to have demonstrated a sustained interest in mathematics and a high level of mathematical ability. The required background includes a full calculus sequence, basic linear algebra, and advanced calculus or basic real analysis. The search of the participants is nationwide. Participants are selected based on their academic record, recommendation letters from faculty and an essay.

The number of MASS participants varies from year to year with an average of 15 per semester. A part consists of Penn State students but the majority are outsiders. It is interesting to analyze where they come from. For this purpose we divide American universities into four categories: 1) small, mostly liberal arts, schools; 2) state universities (mostly large); 3) elite private universities; 4) Penn State. The statistics over the last 6 years is as follows: about 20% of the participants belong to the first category, about 40% to the second, only 3% to the third and 37% to the forth. One should take into account that some Penn State students are part-time participants (they take one or two courses) but a few of them participate in MASS more than once.

These numbers are, probably, not very surprising (although, we strongly feel, even students from elite schools will significantly benefit from the program). Another piece of statistics: the percentage of women was about 30% (with considerable deviations: in 2000, the ratio was 50/50).

About 70% of MASS graduates went to graduate programs in mathematics (one should keep in kind that some of the recent participants continue their undergraduate studies). The distribution of the graduate schools is very wide. Without providing a comprehensive data, we mention some: Harvard, Cornell, Stanford, Princeton, Yale, University of Chicago, University of Michigan, University of California at Berkeley, University of Wisconsin, Indiana University, University of Utah, University of Georgia, etc. About 15% of MASS graduates chose Penn State as a graduate school.

Here is what Suzanne Lynch, a MASS-96 participant who is about to receive her Ph. D. from Cornell, wrote in an unsolicited letter:

The MASS program has been the best semester of my life. I was immersed in an environment of bright motivated students and professors and challenged as never before. I was pushed by instructors, fellow students and something deep inside myself to work and learn about mathematics, and my place in the mathematical world. I loved my time there, and never wanted to leave. I believe the MASS program helped to prepare me for the rigors of graduate school, academically and emotionally. . . . The MASS program has been very instrumental in opening grad school doors to me, and giving me the courage to walk through them.

Talking of MASS participants, one must mention the teaching assistants involved. TA's are chosen from the most accomplished Ph.D. students of the Penn State Department of Mathematics. Their work is quite demanding but also rewarding. TA's are required to sit in the respective class and take notes; once a week they have an 1 hour meeting with the students devoted to problem solving, project discussion and, sometimes, individual tutoring. In some cases the material of a MASS course may be new for the TA as well. This gives the assistant a welcome opportunity to learn a new topic but makes his work even more challenging. Some MASS teaching assistants are themselves MASS graduates.

Students' research

During the semester, each MASS participant works on three individual projects. Usually such a project consists in learning a certain topic in depth, working on exercises and problems (often, quite hard) and making a presentation during the final examination. For many MASS participants who also attend the REU program a project is a continuation of that started in summer.

In some cases, a research project produced a significant piece of mathematical research. Here are two examples:

An Nguyen, a MASS-96 student and now a graduate student in Computer Science at Stanford, rediscovered the famous value of $\lambda = 1 + \sqrt{8}$ for the appearance of period three orbits in the logistic family $f(x, \lambda) = \lambda x(1 - x)$, and then went on to discover a previously unknown bifurcation point where the second period four orbit appears: $\lambda = 1 + \sqrt{4 + \sqrt[3]{108}}$.

James Kelley, a MASS-98 participant now a graduate student at UC Berkeley, studied the representation of integers by quadratic forms, a classical problem in number theory. In particular, he studied a well known problem due Irving Kaplansky: What integers are of the form $x^2 + y^2 + 7z^2$ where x, y, and z are integers. Obviously, if N is of this form, then so is Nk^2. However, the converse if not necessarily true. James proved, using the theory of elliptic curves and modular forms, that every "eligible" integer N which is not a multiple of 7 and not of this form, is square-free! This result has appeared in print: J. Kelley, "Kaplansky's ternary quadratic form", Int. J. Math. Sci. 25 (2001), 289–292.

The research project topics may be related to the student's major, different from mathematics. For example, a biology major in the 2001 course “Mathematical analysis of fluid flow” has a research project “A mathematical analysis of fluid flow through the urinary system”.

MASS students present their research projects at the Undergraduate Student Poster Sessions at the January AMS/MAA joint meetings. For example, N. Salvaterra and B. Wickman (REU and MASS 1999) were among winners in Washington D.C., January 2000 with the poster “The Growth of Generalized Diagonals in a Polygonal Billiard”, advisors: A. Katok and M. Guysinsky. Another example: B. Chan (REU and MASS 2000) was a winner in New Orleans, January 2001 with the poster “Estimation of the Period of a Simple Continued Fraction”, advisors: R. Vaughan and M. Guysinsky.

Funding

MASS is jointly funded by Penn State and NSF. Penn State provides fellowships for out-of-state students that reduce their tuition to the in-state level. Further support comes through the National Science Foundation VIGRE grant. In particular, MASS participants whose tuition in their home institution is lower than Penn State in-state tuition receive grants for the difference. The balance of the VIGRE funds is used to further decrease out-of-pocket expenses of the participants and is distributed individually based on merit and need. In particular, a number of merit fellowships is awarded at the end of MASS semester. The VIGRE grant also supports the MASS colloquium series by covering the speakers’ travel expenses.

Perspectives

We are confident that MASS will continue to grow. Here are some ideas for the program’s future.

- One of the key issues is funding. We hope to attract private money to complement the current NSF support of the program. There is a considerable interest in mathematics among private and corporate donors, and the contribution of MASS program to undergraduate mathematical education is quite substantial. Ideally, we would like to see the whole program endowed.
- We envision a larger, 2-level MASS program that runs two consecutive semesters, one is oriented toward freshmen and sophomores, the other, more advanced, for juniors and seniors.
- With a broader financial base, MASS could include a certain number of foreign students. The available NSF funds can support only US citizens and permanent residents. However, there is an interest in the program among foreign students already

in American universities, and a small number of such students has attended MASS paying from their own funds. As a first step, we would like to extend the program to undergraduates in Canada.

- An important issue is preservation of MASS materials. Each MASS core course, developed for the program, can be used elsewhere. We envision an ongoing series of small books containing course material in a lecture notes style, detailed enough for a qualified instructor to design a similar course and to serve as guidelines. As a first step, we are preparing a MASS presentation volume that will be published by the American Mathematical Society. This book will present all components of the program (core courses, REU courses, MASS colloquia, students' research), and it will appear late in 2002 or early in 2003. We also hope to record MASS colloquium talks and make them available to public, possibly, online, in the MSRI style.

Our optimism about the future of MASS is based on the enthusiasm of the students who attend the program, the instructors and TA's who teach the classes, and on the general public interest in improving the mathematical education in the USA.

Appendix 1: On the Russian tradition of mathematical education

It is a common knowledge that Russian mathematics constitutes one of the most vital and brilliant mathematical tradition of the 20th century. Moreover, mathematicians, trained in Russia are currently very well represented in the top echelon of the world mathematical community. Behind this flourishing stands a powerful tradition of spotting and training mathematical talent, which is not without its downsides. The subject is certainly too complex for a detailed discussion, but we will try to present a brief outline.

A typical path of a mathematically talented student would start rather early. It would include participation in mathematical olympiads of various levels, from school district to the all-Union one (the first Mathematical Olympiad in Soviet Union was the Leningrad one in 1934, and Moscow followed the suit the next year; the first all-Union Olympiad took place in 1961). Another activity for an interested school student was a *kruzhok* (literally, "circle"; a closer English equivalent is, probably, "workshop"); *kruzhki* also appeared in the mid-30's. They usually met at the university once a week in the evening and were run by dedicated undergraduate or graduate students with a tremendous enthusiasm for mathematics, very often, themselves alumni of a *kruzhok* – a good example of "vertical integration"! The material discussed usually went well beyond the secondary school curriculum and included challenging problems and nonstandard topics from elementary to higher mathematics.

Beginning in the early 1960's, special high schools for mathematics and physics were organized in major cities. Many benefited from the help of the local university faculty. For example, E. B. Dynkin and I. M. Gelfand played a prominent role in running the legendary Moscow School No 2, whose many alumni are now professors of mathematics in universities across the globe. Another well-known high school, the Boarding School for Mathematics No 18 at Moscow State University, was established by A. N. Kolmogorov. Unlike other mathematical schools in Moscow which essentially sprang from private initiative and had no special funding, this school was a special institution affiliated with the university and specially funded by the state. Still other celebrated Moscow schools for mathematics were No 7, No 57 and No 444 (the second and third authors are alumni of these schools, No 7 and 2, respectively, and the first and the third authors taught in School No 2). The mathematical curriculum of a special school was more intense and systematic than that of the *kruzhki*, and this influenced our thinking about the structure of the MASS program.

An essential part of the tradition was participation of prominent mathematicians of various ages in teaching and popularizing mathematics. A typical example is the "Kvant" (means "Quantum") magazine on physics and mathematics for school students published since 1970. Kvant had 12 issues a year and, in the peak of its popularity in the mid-70's, boasted more than 300 000 subscribers. Among the authors, one finds the names of well known mathematicians: A. D. Alexandrov, V. I. Arnold, D. B. Fuchs, I. M. Gelfand, S. G. Gindikin, A. A. Kirillov, A. N. Kolmogorov, M. S. Krein, Yu. V. Matiyasevich, S. P. Novikov, L. S. Pontryagin and many others. For many generations of students, Kvant opened new horizons and determined their choice of mathematics as a profession. Along with Kvant, there existed a rich popular literature, from numerous collections of problems for all ages to books on various topics in "serious" mathematics. We would like to mention some people who made a very substantial contribution to popularization of mathematics: N. B. Vassiliev, N. Ya. Vilenkin, I. M. Yaglom. The third author of this article worked, for a number of years, as the Head of Kvant's Mathematics Department.

At the university level, the emphasis on creative thinking has continued, sometimes to the detriment of the systematic learning. For example, the standard mandatory courses often did not fully reflect the most current thinking in their subjects, and were looked down upon by the top students. A very important role was played by topics courses, offered in a wide variety of subjects and attended by a mixture of undergraduate and graduate students. Similarly, specialized seminars were usually attended by a mix of undergraduates, graduates and established mathematicians. Starting from the third year of the university every student had an advisor and hence was considered as a member of a research community in his/her field. It was not unusual for best undergraduate students at major universities to have papers published in first rate research journals by the end of the 5 years of their undergraduate studies.

This system had multiple effects. On the one hand, it stimulated early development of research interests and mathematical precocity. On the other hand, it lead to often inflated standards and expectations and, eventually, to a great wastage of talent. A typical victim of the system was a student with a considerable talent but not very high self-esteem (often female). Still, the overall results were quite spectacular in producing a large number of exceptionally creative and technically powerful mathematicians.

Appendix 2: MASS courses

What follows is a list of MASS courses and the instructors.

2002:

Elements of Fractal Geometry and Dynamics
by Ya. Pesin.

Number Theory with a Tilt at Elliptic Curves
by D. Brownawell.

Intuitive Topology
by S. Tabachnikov.

2001:

Mathematical Analysis of Fluid Flow
by A. Belmonte.

Combinatorics
by G. Andrews.

Geometry and Relativity: An Introduction
by N. Higson.

2000:

Real and p-Adic Analysis
by S. Katok.

Finite Groups, Symmetry and Elements of Group Representations
by A. Ocneanu.
Projective and Non-Euclidean Geometry
by V. Nistor.

1999:
Mathematical Methods in Mechanics
by M. Levi.
Topics in Number Theory
by R. Vaughan.
Geometric Structures, Symmetry and Elements of Lie Groups
by A. Katok.

1998:
Functions and Dynamics in One Complex Variable
by G. Swiatek.
Number Theory: From Fermat's Little Theorem to his Last Theorem
by K. Ono.
The Exponential Universe
by J. Roe.

1997:
Real Analysis
by N. Higson.
Arithmetic and Geometry of the Unimodular Group
by S. Katok.
Explorations in Geometry
by D. Burago.

1996:
Introduction to Dynamical Systems
by A. Katok.
Explorations in Number Theory
by G. Andrews.
Linear Algebra in Geometry
by V. Nistor.

Appendix 3: MASS Colloquia

Here is a list of MASS colloquium talks. The affiliations of the speakers is given as of the time of the talk.

2001:
Family Algebras
by A. Kirillov, University of Pennsylvania.
Integral Lexicographic Codes

by J. Conway, Princeton University.
Klein's Erlanger Programme
by A. Banyaga, Penn State University.
Virasoro group and projective structures on the circle: a hidden face of an old coin
by V. Ovsienko, Centre of Theoretical Physics, Luminy, France.
Vector Calculus and the Topology of Domains in 3-space
by H. Gluck, University of Pennsylvania.
Minimal surfaces, discrete harmonic analysis, and random walks
by A. Sossinsky, Independent University of Moscow, Russia.
Physical proofs of mathematical theorems
by M. Levi, Penn State University.
One hundred years of Hilbert's fourth problem
by J. C. Alvarez, New York Polytechnic University.
Loops in R3: New Angles on an Old Topic
by B. Solomon, Indiana University.

2000:
Knotted Flowlines
by R. Ghrist, Georgia Institute of Technology.
Euler, Jacobi, Ramanujan: Interesting Formulae, Beautiful Results and Some Recent Variations
by I. Kra, State University of New York at Stony Brook.
Aperiodic Tilings and Computation in the Hyperbolic Plane
by Ch. Goodman-Strauss, University of Arkansas.
Search Engines and Measurements on the Modular Group
by L. Polterovich, Tel Aviv University, Israel.
Internet Search and Markov Chains
by M. Brin, University of Maryland .
The Peano Kernel: Convolving Abstract and Applied Mathematics
by M. Gage, University of Rochester.
Flexible Polyhedra
by D. Fuchs, University of California at Davis.
The Story of the Rogers-Ramanujan Identities
by G. Andrews, Penn State University.
The Future of Mathematics
by D. Zeilberger, Temple University.

1999:
The Meaning of Chaos
by G. Swiatek, Penn State University.
When Do Polynomials Have Common Zeros?
by D. Brownawell, Penn State University.

Knots and Knot Invariants
by A. Vaintrob, University of New Mexico.
Hilbert's 16th Problem Near Its Centenary
by Yu. Ilyashenko, Cornell and Independent University of Moscow.
Replicating Tiles
by V. Nitica, Notre Dame University.
Nightmares and Dreams of Lyapunov : Stability and Semigroups
by Yu. Latushkin, University of Missouri.
On the Hilbert-Smith Conjecture
by A. Dranishnikov, Penn State University.
The DNA Geometric Inequality: An Open-ended Story
by S. Tabachnikov, University of Arkansas.
The Banach-Tarski Paradox and Amenable Groups
by M. Guysinsky, Tufts University.
Group Reconstruction
by C. Plaut, University of Tennessee.

1998:
Rational numbers, right triangles, and elliptic curves
by K. Rubin, Stanford University.
Bicycle wheels, Gauss-Bonnet formula and Berry's phase
by M. Levi, Penn State University.
Why representation theory is interesting and useful?
by A. Kirillov, University of Pennsylvania.
Playing pool with pi
by G. Galperin, Eastern Illinois University.
The Life and Work of India's Greatest Mathematician Srinivasa Ramanujan
by B. Berndt, University of Illinois.
Mathematics of Fractal Images
by Ya. Pesin, Penn State University.
An Introduction to K-theory
by P. Baum, Penn State University.
Arithmetic Curves and Triangulations
by F. Bogomolov, Courant Institute.
Social Life of Curves
by S. Gindikin, Rutgers University.

1997:
Connecting the dots: the theory and practice of interpolation
by D. Arnold, Penn State University.
Why the symplectic and contact geometries are so important?
by A. Kirillov, University of Pennsylvania.

Mountaineering and quantum mechanics
by J. Roe, Oxford University, UK.
Platonic solids, their symmetry groups and Klein's invariant theory
by A. Ocneanu, Penn State University.
Probabilistic strategies in games
by S. Ivanov, Steklov Institute for Mathematics, Russia.
Invariants of curves by evolution of frames
by R. Bishop, University of Illinois at Urbana-Champaign.

1996:
The man who loved numbers, Biography of Ramanujan
by G. Andrews, Penn State University.
Stability of the inverted pendulum and topology of the symplectic group
by M. Levi, Rensselaer Polytechnic Institute.
On classification of the finite simple groups
by E. Formanek, Penn State University.
Dynamics of the logistic family
by G. Swiatek, Penn State University.
Playing billiard with a help of mathematics
by Yu. Suhov, University of Cambridge, UK.
Unprovable theorems and fast-growing functions
by S. Simpson, Penn State University.

Appendix 4: REU courses

2002:
Partitions and the Omega Package
by G. Andrews
Differential Equations As Dynamical Systems
by Ya. Pesin

2001:
Continued Fractions, Hyperbolic Geometry, and Quadratic Forms
by S. Katok
Galois Theory
by V. Nistor

2000:
Dynamical Systems and Applications
by M. Levi
The Prime Numbers and Their Properties
by R. Vaughan

1999:

Symmetry and Representations of Finite Groups
  by A. Ocneanu
The Theory of Partitions
  by G. Andrews

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