Mad about MASS!

My name is Sharon Chuba; I am a junior majoring in Mathematics at Penn State (University Park). I am from Downingtown, Pennsylvania, and graduated from Downingtown High School in 1999. While in high school, I decided I wanted to be a university math professor, because math is fun. I chose Penn State so I could have a well-rounded undergraduate education. When I told my parents that I planned to pursue a degree in math, they fully supported my decision, but also suggested that I might like studying related courses, such as computer science and economics. I chose a minor—in Psychology.

After two years of loving my math classes at Penn State, I learned about the MASS program through my adviser, Professor Diane Henderson, and was encouraged to participate by one of my instructors, Professor Bryna Kra. I was a little doubtful that I would be able to handle such an intense mathematics program, but I finally decided to attend for two reasons. First, I needed to make sure that I liked mathematics enough to devote my entire graduate education to its study. Second, as a student in the Schreyer Honors College, I had to obtain 14 honors credits during my junior and senior years. The MASS program offered 16 honors credits in one semester.

I am really glad I chose to attend MASS! This year there were ten full-time students, from as far south as Florida and as far west as California, and five Penn State students (who participated part-time) in the program. The MASS Program consists of three core courses in algebra, analysis, and geometry; each focusing on a topic generally not studied by undergraduates. My courses were MATHEMATICAL ANALYSIS OF FLUID FLOW with Professor Andrew Belmonte, COMBINATORICS with Professor George Andrews, and GEOMETRY AND RELATIVITY with Professor Nigel Higson.

As part of these courses, each student completes a research project to be presented during the oral final exam. These projects are a chance for students to research topics of interest not covered by the course, or try new research. There are weekly two-hour seminars discussing “everything all good mathematicians know, but don’t know where they learned it” (Professor Sergei Tabachnikov), and one-hour colloquia featuring speakers from inside and outside of Penn State. (I especially liked the colloquia because our program assistant, Flossie Dunlop, would bake cookies.) MASS participants were also invited to the many other talks held by the department.

Though I had been hesitant about participating at first, within the first week I knew I was going to love this program. Suddenly I had quickly paced classes where I got to see connections between each field of mathematics, instead of taking individual algebra, analysis, and geometry courses without any overlap. I loved the program not only for the uniqueness of the classes, but also for the opportunity to interact with other math fanatics. The weekly MASS colloquia and seminars were a chance to hear about current topics of interest in mathematics, and meet professors and graduate students in Penn State’s math department. I also heard about opportunities for undergraduate math majors—conferences and REUs (research experiences for undergraduates, usually summer programs), and the Putnam exam, a math problem-solving competition given yearly at colleges and universities across the United States.

I had been introduced to mathematical research during Penn State’s summer REU the summer before I entered the MASS program, so I wanted at least one of my projects to be new research. After meeting with each professor to discuss topics that I would not only enjoy, but also be capable of studying, I decided to do two new research projects, while for my geometry course, I researched Morley’s theorem. This is a nifty little discovery at the turn of the 20th century stating that, given an arbitrary triangle, the pair-wise intersections of the angle trisectors form the vertices of an equilateral triangle. I was most interested in the fact that by 1970 there were over 150 known proofs of this theorem.

For my fluids project, Professor Andrew Belmonte had suggested I look at some of the equations we had studied during class, such as Euler’s equation, which links the velocity field of a fluid to the gradient of the pressure on the fluid and external forces acting on this fluid, and consider what would happen to these equations when we assume there are no external forces or pressure. I looked specifically at incompressible Euler’s equation. Andrew Belmonte is extremely enthusiastic and kept me interested in my project. I would work for a little bit, become stuck and a little less motivated, show my results to Andrew, and then he would make suggestions and get me excited again. After a few of these cycles, I showed Andrew a few general solutions, and he noticed that all of the time dependent flows I found were interesting, but the steady state solutions I found were all rather boring. He then conjectured that all of the steady state solutions would indeed be boring. This kept me occupied for a large portion of the project, until I suddenly had a burst of inspiration during the last 10 minutes of the last class of the program. This thought led to a simple mathematical proof of the “boring” conjecture. Andrew Belmonte and I are currently working on writing up our results for possible publication.

While I enjoyed every class, combinatorics was by far my favorite and resulted in my favorite project. Partitions, in its most basic form, is the study of how to write a

TA Dmitri Matsnev, Sharon Chuba, and Andrew Belmonte

number as the sum of smaller numbers. This may sound trivial, but it is actually very exciting and useful. During the class, we had studied MacMahon's Omega function, which is a way of inputting conditions on the parts we wanted in our partitions, and receiving back the generating function for the corresponding class of partitions. In addition, Professor Andrews introduced us to the Omega package for Mathematica, a software program he and two other mathematicians had developed to facilitate these computations. As an example of the Omega function, we considered all of the triangles with sides of whole number length and perimeter n. To see how many triangles corresponded to each n, we gave the triangle inequality to Omega as input, and we received back the generating function for a partition of n into 2s, 3s, and 4s so that 3 appears at least once. Once we found this correspondence, we explained it by finding a bijection between these partitions and triangles.

I wanted a project where I could look for new results while practicing my computer skills with the Omega package. At first I thought that if triangles gave such an easy result, maybe I should consider polygons, until Professor Andrews pointed out that triangles are rather rigid and restrictive with the triangle inequality, whereas polygons are not. After a few modifications, I decided to look for a correspondence between partitions of n and n as the sum of the sides of κ triangles glued together such that the longest side of each triangle was glued to the shortest side of the previous triangle. Case $k=1$ is the single triangle we considered in class. This turned out to be a non-trivial problem. I eventually had to consider three cases, and found results for two of these three cases. I plan to continue my research on the remaining case.

My research lead to unexpected opportunities to travel to exotic places. In January 2002, I presented my partitions project in the Undergraduate Poster Session of the Joint AMS-MAA meeting in San Diego. Because I was presenting my work at the meeting, MASS funded my travel expenses. This was the first mathematics conference I had ever participated in, and I really enjoyed attending the talks. As an added bonus, I won a first place award for my poster.

In February 2002, I attended the 4th annual Nebraska Conference for Undergraduate Women in Mathematics and presented a 15-minute talk on my fluids project. Sergei Tabachnikov always told us that our MASS colloquium talks should be accessible for at least the first ten minutes. As my talk was only 15 minutes long, it mostly consisted of defining the terms in my title, "Pressureless Incompressible Euler Flow in 2D with No External Forces." Nonetheless I enjoyed my first chance to speak about my work to a large audience.

Participating in the MASS program was in some ways a humbling experience, and in other ways a confidence-builder. I met a lot of other serious (but fun) math students (math fanatics) with varying backgrounds and experiences, some of whom I had to ask to help me understand a few of the class problems. I learned that if I put in a lot of hard work and effort, I was capable of understanding difficult material and conducting research. Unfortunately, MASS is only a semester long. I miss total immersion in mathematics. I definitely know now that I want to attend graduate school to study mathematics. At least I know that when fall 2003 rolls around, I can attend the weekly MASS colloquia, and enjoy more of Flossie's baking!

For more information about:
1. Morley's theorem, try www.cut-the-knot.com for some interesting history and proofs.
2. mathematical fluid flow, try Richard E. Meyer's *Introduction to Mathematical Fluid Dynamics* (1971).
3. partitions, try George Andrew's *Theory of Partitions* (1984).
4. MASS, try www.math.psu.edu/MASS