Critical Thresholds in Eulerian Dynamics

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We study the questions of global regularity vs. finite time breakdown in Eulerian dynamics , $\partial_t U + U \cdot \nabla_x U = \nabla_x F$, which shows up in different contexts dictated by different modeling of F's. To address these questions, we propose the notion Critical Threshold (CT), where a conditional finite time singularity depends on whether the initial configuration crosses an intrinsic, $O(1)$, critical threshold. Our approach is based on a main new tool of spectral dynamics, where eigenvalues, $\lambda = \lambda(\nabla_x U)$, and eigenpaairs (l, r), are traced by the forced Ricatti equation $\partial_t \lambda + U \cdot \nabla_x \lambda + \lambda^2 =< l, D^2 F r >$. We shall outline three prototype models. We begin with the $n-$dimensional Restricted Euler equations, obtaining $[n/2]+1$ global material invariants which precisely characterize the local topology at breakdown time. Next we introduce the corresponding $n-$dimensional Restricted Euler-Poisson (REP)system, identifying a set of $[n/2]$ global invariants, which yields a remarkable characterization of 2D initial REP configurations with global smooth solutions. And finally, we identify the CTs for models with rotational forcing. Our study reveals the dependence of the CT phenomena on the initial spectral gap, $\lambda_2(0) - \lambda_1(0)$.