M598B: Hints to Homework Assignment 8

Date: Oct. 23, Tuesday

1. See lecture notes.

2. Use the definition of the Fourier transform. The integral is actually easy to evaluate. Just use the fact that

$$\int e^{\alpha x} dx = \frac{1}{\alpha} e^{\alpha x} + C$$

even when α is a complex number.

3. Use definition of Fourier transform and change of variables of the integration.

4. Follow the example of the lecture notes and use the previous shift theorem.

5. Use definition of Laplace transform and integration by parts.

6. Remember the variable x is the same as t. The equation is an ordinary differential equation. In the latest version of the homework, I have changed x to t.

Use Laplace transform. Transform the right-hand side e^{-t} also. Use a formula for that from the lecture notes.

In version, use this formula:

$$\mathcal{L}^{-1}[\frac{1}{(s-a)^n}](t) = \frac{t^{n-1}}{(n-1)!} e^{at}.$$

This is a common formula which is available in most text books. I am adding this formula to the lecture notes.

7. For partial fraction of

$$\frac{s-2}{s^2-s-1},$$

first factorize the denominator:

$$s^2 - s - 1 = (s - s_1)(s - s_2),$$

where s_1 and s_2 are the two roots. Then propose the form

$$\frac{s-2}{s^2-s-1} = \frac{\alpha}{s-s_1} + \frac{\beta}{s-s_2}.$$

Multiply the equation with $(s - s_1)(s - s_2)$, compare coefficients to find a couple of equations for α and β. The final answer involves $\sqrt{5}$ and exponentials.