M598B: Homework Assignment 8

Date: Oct. 15, Monday Due Wed. Oct. 24.

1. Follow the proof of the Fourier Inversion Theorem, prove (formally) the Parseval (or Plancherel)Identity:

$$\|\hat{f}\|_{L^2(\mathbb{R}^1)} = \|f^{\vee}\|_{L^2(\mathbb{R}^1)} = \|f\|_{L^2(\mathbb{R}^1)}.$$

(I might have accidently provided this proof in the typed lecture notes, good luck.)

2. Find the Fourier transform of

$$f(x) = \begin{cases} 0, & |x| > a \\ 1, & |x| < a. \end{cases}$$

3. Show that the Fourier transform of $f(x-c)$ is $e^{i\mu c}\hat{f}(\mu)$. (This is called the shifting theorem.)

4. Use Fourier transform to solve the heat (diffusion) equation with convection:

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} + c\frac{\partial u}{\partial x}, \quad (x \in \mathbb{R}^1)$$

where $k > 0$, with initial condition

$$u(0,x) = f(x).$$

5. Find the Laplace transform of the function $f(t) = t^3, t \geq 0$.

6. Use Laplace transform to solve

$$\begin{aligned} u'' + 2u' + u &= e^{-t} \quad \text{for } t > 0, \\ u(0) &= 0, \\ u'(0) &= 0. \end{aligned}$$

7. Use Laplace transform to solve

$$\begin{aligned} u'' - u' - u &= 0 \quad \text{for } t > 0, \\ u(0) &= 1, \\ u'(0) &= -1. \end{aligned}$$

(Seven problems in all, about 14.4 points each.)