M598B: Homework Assignment 4

Date: September 10, Wed, 2001. Due Wed. September 19.

1. Form a scalar by contracting the tensor with the matrix

$$\begin{pmatrix} 5 & 0 & 1 \\ 3 & 6 & 3 \\ 4 & 5 & 4 \end{pmatrix}.$$

2. Given that

$$(T_{ik}) \quad = \quad \begin{pmatrix} 1 & 0 & 2 \\ 3 & 6 & 3 \\ 4 & 2 & 4 \end{pmatrix}, \qquad \mathbf{A} = \mathbf{i}_1 + 2\mathbf{i}_2 + 3\mathbf{i}_3.$$

Find the inner products $T_{ik}A_i$ and $T_{ik}A_k$. (Hint: First index in T is row number.)

3. Let T_{ik} and $\mathbf{A}$ be as in the preceding problem (problem 2), and let

$$\mathbf{B} = 4\mathbf{i}_1 + 5\mathbf{i}_2 - \mathbf{i}_3.$$

Find the inner product $T_{ik}A_iB_k$.

4. Prove that if S_{ik} is a symmetric tensor and A_{ik} an antisymmetric tensor, then $S_{ik}A_{ik} = 0$.

5. Let T_{ik} be as in problem 2. Find the symmetric part S_{ik} and the antisymmetric part A_{ik} of the tensor T_{ik}.

6. Given a scalar function $\Phi = \Phi(x_1, x_2, x_3)$, do the quantities

$$\frac{\partial^2 \Phi}{\partial x_i \partial x_k}$$

form a tensor?

7. The unit base vectors $\mathbf{i}'_i$ of a new coordinate system K' are given by

$$\mathbf{i}'_1 = \frac{\mathbf{i}_2 + \mathbf{i}_3}{\sqrt{2}}, \ \mathbf{i}'_2 = \frac{\mathbf{i}_1 - \mathbf{i}_2 + \mathbf{i}_3}{\sqrt{3}}, \ \mathbf{i}'_3 = \frac{2\mathbf{i}_1 + \mathbf{i}_2 - \mathbf{i}_3}{\sqrt{6}}.$$

The stress tensor p_{ik} in the system K is of the form

$$(p_{ik}) \quad = \quad \begin{pmatrix} p_1 & 0 & 0 \\ 0 & p_2 & 0 \\ 0 & 0 & p_3 \end{pmatrix}.$$

Find the components p'_{11} and p'_{12} of the stress tensor p'_{lm} in K'.

8. Let a_i, b_j, and c_k be the components of three vectors. Verify that the 27 quantities

$$d_{ijk} = a_i b_j c_k$$

form a tensor of order 3.

9. The stress tensor at a point has components given by

$$(s_{ij}) \quad = \quad \begin{pmatrix} 2 & -1 & 2 \\ -1 & 3 & 0 \\ 2 & 0 & -1 \end{pmatrix}.$$

Find the stress vector $(\mathbf{p}_n)$ across an area normal to the unit vector

$$\mathbf{n} = (\mathbf{i}_1 - \mathbf{i}_2 + \mathbf{i}_3)/\sqrt{3}.$$

What is the normal stress across such an area (i.e, the projection $(\mathbf{p}_n \cdot \mathbf{n})\mathbf{n}$ of the vector $\mathbf{p}_n$ on to $\mathbf{n}$)?

10. The formula

$$C_j = \epsilon_{jkl} A_k B_l$$

gives the components of the vector product $\mathbf{C} = \mathbf{A} \times \mathbf{B}$. Verify this by calculating the second component C_2.