M598B: Hints to Homework Assignment 2

3. Evaluate the line integral

$$\int_L \mathbf{C} \cdot d\mathbf{r}$$

where $\mathbf{C} = (x_2, -x_1, -1)$ and L is a directed curve given by the graph of the vector $\mathbf{A}(t)$ in Exercise 1 from $t = 0$ to $t = 2\pi$.

Hints: Note that

$$\mathbf{C} \cdot d\mathbf{r} = x_2 dx_1 - x_1 dx_2 - dx_3.$$

Then use the parametrization given in Exercise 1:

$$\mathbf{C} \cdot d\mathbf{r} = (\sin t)(-\sin t)dt - (\cos t)(\cos t)dt - 2dt.$$

Simplify it and integrate it to finish.

6. Find a unit normal vector to the surface

$$x_3 = 2 - x_1 - x_2^2.$$

hint. Read the class example first. If it does not help much. Try this: Let

$$\phi = x_3 - 2 + x_1 + x_2^2.$$

Take the gradient of ϕ. Normalize it. For normalization, see Lecture one.

8. Find the total flux of the vector field $\mathbf{A} = (x_1, x_2, x_3)$ out of the unit sphere: $x_1^2 + x_2^2 + x_3^2 = 1$.

hint. Do not hesitate in using the definition of flux, the just-learned way of finding a unit normal of a surface, then the calculation is actually very simple. Remeber that the surface integral over a surface S with the integrand 1 is simply the surface area of S.

10. Let

$$\mathbf{F}(x_1, x_2, x_3) = (x_1\mathbf{i}_1 + x_2\mathbf{i}_2 + x_3\mathbf{i}_3)/r^3$$

where $r^2 = x_1^2 + x_2^2 + x_3^2$. Show that the flux of this vector through any closed surface S is 0 if the origin is not enclosed by S.

Hint: Gauss' Theorem.