M598B: Homework Assignment 14 (Last One)

Date: Nov. 26, Monday; Due Wed. Dec. 5.

1. Use Rayleigh quotient (Section 6.12, Conclusion No. 6) to show that any eigenvalue must be positive ($\lambda > 0$) for Bessel's equation

$$\begin{cases} (ru')' - \frac{m^2}{r}u + \lambda r u = 0, \quad 0 < r < a, \\ u(a) = 0, \\ |u(0)| < \infty, \ |u'(0)| < \infty. \end{cases} \tag{1}$$

(Hint: Once you use the Rayleigh quotient and the boundary conditions, the positivity of λ_n is obvious.)

2. Use separation of variables to solve the two-dimensional eigenvalue problem for the Laplacian in a rectangle

$$\begin{cases} u_{xx} + u_{yy} + \lambda u = 0, \quad \text{in } 0 < x < L, \ 0 < y < H, \\ u = 0 \text{ on the boundary.} \end{cases} \tag{2}$$

(The eigenfunctions should be the eigenfunctions of Section 6.9.)

3. Solve the heat equation in a rectangle

$$\begin{cases} u_t = k\Delta u, \ \text{in } 0 < x < L, \ 0 < y < H, \\ u(0, x, y) = g(x, y), \\ u(t, x, y) = 0 \text{ on the boundary of the rectangle.} \end{cases} \tag{3}$$

(This was intended as Section 6.10.4.)

END