

# M598B: Homework Assignment 13

Date: Nov. 19, Monday; Due Wed. Nov. 28.

1. Find a solution to

$$\begin{aligned}\frac{\partial^2 u}{\partial t^2} &= c^2 \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right), & 0 < x < L, \quad 0 < y < H \\ u &= 0 & \text{on the boundary of the rectangle,} \\ u(0, x, y) &= \sin\left(\frac{10\pi x}{L}\right) \sin\left(\frac{5\pi y}{H}\right), \\ \frac{\partial u}{\partial t}(0, x, y) &= 0.\end{aligned}$$

2. Use eigenfunction expansion to derive a solution to

$$\begin{aligned}\frac{\partial^2 u}{\partial t^2} &= c^2 \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + Q(t, x, y), & 0 < x < L, \quad 0 < y < H \\ u &= 0 & \text{on the boundary of the rectangle,} \\ u(0, x, y) &= 0, \\ \frac{\partial u}{\partial t}(0, x, y) &= 0.\end{aligned}$$

3. Use eigenfunction expansion to derive a solution to

$$\begin{aligned}\frac{\partial u}{\partial t} &= \frac{\partial}{\partial x} \left( K(x) \frac{\partial u}{\partial x} \right) + Q(t, x), & 0 < x < L, \\ u(t, 0) &= 0, \\ u(t, L) &= 0, \\ u(0, x) &= 0.\end{aligned}$$

Here assume that  $K(x) > 0$  and smooth. The eigenfunctions do not need to be explicit.