M598B: Homework Assignment 12

Date: Nov. 12, Monday; Due Wed. Nov. 21.

1. Use separation of variables to derive a solution formula for

$$\begin{aligned} \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} &= 0, \qquad 0 < x < L,\ 0 < y < H \\ u(0,y) &= 0, \\ u(L,y) &= 0, \\ u(x,0) &= 0, \\ u(x,H) &= g(x). \end{aligned} \tag{1}$$

2. By guessing or eigenfunction expansion, find a solution to

$$\begin{aligned} \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} &= \sin\frac{2\pi x}{L}\sin\frac{\pi y}{H}, \qquad 0 < x < L,\ 0 < y < H \\ u(0,y) &= 0, \\ u(L,y) &= 0, \\ u(x,0) &= 0, \\ u(x,H) &= 0. \end{aligned} \tag{2}$$

3. Use separation of variables to derive a solution formula for

$$\begin{aligned} \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} &= 0, \qquad 0 < x < L,\ 0 < y < H \\ \frac{\partial u}{\partial x}(0,y) &= 0, \\ \frac{\partial u}{\partial x}(L,y) &= 0, \\ u(x,0) &= 0, \\ u(x,H) &= g(x). \end{aligned} \tag{3}$$

(You may use Fourier cosine series.)

4. Use change of variables to reduce

$$\begin{aligned} \frac{\partial u}{\partial t} &= k\frac{\partial^2 u}{\partial x^2} + Q(t,x), \qquad 0 < x < L, \\ u(0,x) &= g(x), \\ \frac{\partial u}{\partial x}(t,0) &= A(t), \\ \frac{\partial u}{\partial x}(t,L) &= B(t) \end{aligned} \tag{4}$$

to a problem with homogeneous boundary condition.

End.