

# MATH 580: Homework Assignment 1

Date: August 29, 2001. Due Tuesday September 6.

1. Prove that the projection of a sum of vectors onto any axis equals the sum of the projections of the vectors onto the same axis.
2. A parallelogram has acute angle  $\pi/3$  and side lengths  $a = 3, b = 5$ . Thinking of the corresponding sides as vectors  $\mathbf{a}$  and  $\mathbf{b}$  find
  - (a) The vectors  $\mathbf{a} + \mathbf{b}$  and  $\mathbf{a} - \mathbf{b}$  (what is their geometric meaning?); (No coordinate system needed.)
  - (b) The area of the parallelogram.
3. Given the vectors

$$\begin{aligned}\mathbf{A} &= \mathbf{i}_1 + 2\mathbf{i}_2 + 3\mathbf{i}_3, & \mathbf{B} &= 4\mathbf{i}_1 + 5\mathbf{i}_2 + 6\mathbf{i}_3, \\ \mathbf{C} &= 3\mathbf{i}_1 + 2\mathbf{i}_2 + \mathbf{i}_3, & \mathbf{D} &= 6\mathbf{i}_1 + 5\mathbf{i}_2 + 4\mathbf{i}_3;\end{aligned}$$

where  $\mathbf{i}_1, \mathbf{i}_2, \mathbf{i}_3$  are an orthonormal basis. Find

- (a)  $\mathbf{A} + \mathbf{B} - \mathbf{C}$ ;
  - (b)  $\mathbf{A} \cdot \mathbf{B}$ ;
  - (c) The angle made by  $\mathbf{C}$  and  $\mathbf{D}$ ;
  - (d) The projection of  $\mathbf{A}$  onto the direction of  $\mathbf{B}$ ;
  - (e) The vector product  $\mathbf{A} \times \mathbf{B}$ .
4. Show that the four vectors  $\mathbf{A}, \mathbf{B}, \mathbf{C}$ , and  $\mathbf{D}$  are linearly dependent.
  5. Verify the following identity:

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) + \mathbf{b} \times (\mathbf{c} \times \mathbf{a}) + \mathbf{c} \times (\mathbf{a} \times \mathbf{b}) = \mathbf{0}.$$