

# Review2 for math22

November 10, 2006

## 1, What is a Function(3.1)

(1) The definition of a function

Example:  $\{(1, 2)(3, 4)(5, 2)\}$ ,  $(x - 1)^2 + (y + 3)^2 = 1$ ,  $|y| = x, y^2 = x + 8, y = \sqrt{x + 2} - 1$

(2) Determine the domain of  $f(x) = \frac{\sqrt{x+3}}{\sqrt{x}}$

(3) Function notation:  $f(x)$ ;

Let  $f(x) = \frac{x}{x-4}$ , find  $f(t)$ ,  $f(a^2)$  and  $f(3)$ , find the input if  $f(x) = 3$

## 2, Graphs of Function(3.2)

(1) Definition (page 224), vertical line test.

(2) Determine domain, range and function value from a graph.

(3) Six popular graphs (page 232).

(4) Graph a piecewise function.

## 3, Increasing and Decreasing Functions; Average Rate of Change(3.3)

(1) Increasing and decreasing functions.

(2) Average rate of change of a function on an interval  $[a, b]$ :  $\frac{f(b)-f(a)}{b-a}$ .

(3) Linear functions have constant rate of change.

## 4, Transformations of Functions(3.4)

Go over translations, reflections, stretching and shrinking.

## 5, Quadratic Functions; Maxima and Minima(3.5)

(1) Terminology: parabola, vertex

(2) Complete the square for  $y = ax^2 + bx + c$  and write it into the equivalent form  $y = a(x - h)^2 + k$ .  $(h, k)$  is the vertex.

Example: graph  $y = -2x^2 - 4x + 8$ .

(3) Maximum or minimum of a quadratic function occurs at  $x = -\frac{b}{2a}$ . If  $a > 0$ , then the minimum is  $f(-\frac{b}{2a})$ . If  $a < 0$ , then the maximum is  $f(-\frac{b}{2a})$ .

Example: compute the maximum or minimum values of  $f(x) = x^2 - 6x + 73$ ,  $g(x) = x^4 - 6x^2 + 73$ .

## 6, Combining Functions(3.6)

(1) Arithmetical operations with functions.

(2)Composition of functions.

### 7, One-to-one Functions and Their Inverses (3.7)

(1)One-to-one function and horizontal line test.

(2)Property of Inverse functions:  $f(f^{-1}(x)) = f^{-1}(f(x)) = x$

Examples:

1) $f(x) = 4x^3$ , find  $f(f^{-1}(5))$

2)Let  $f(x) = \frac{x-1}{3x+5}$ , find  $f^{-1}$  and its domain.

(2)The graphs of inverse functions are symmetric about the line  $y = x$ .

### 8, Polynomial Functions and Their Graphs(4.1)

(1) $p(x) = a_nx^n + a_{n-1}x^{n-1} + \dots + a_1x + a_0$ , degree

(2)End behavior of graphs of polynomial functions (page 312)

(3)Guidelines for Graphing Polynomial Functions (page 316)

(3)The behavior of a polynomial function near a zero (page 319)

Example: Graph  $y = x^2(x - 1)(x + 2)^2$ .

### 9, Dividing Polynomials(4.2)

(1)Long division of polynomials

(2)Synthetic Division for divisor  $x - c$

(3)The remainder and factor theorems (page 329)

### 10, Real Zeros of Polynomials(4.3)

(1)Rational zeros theorem (page 333)

Find the rational zeros of  $P(x) = x^4 - 5x^3 - 5x^2 + 23x + 10$  by using the above theorem and the synthetic division.

(2)Descartes' rule of signs (page 336)

(3)upper and lower bounds theorem (page 337)

See Example 6 in the book (page 338) for factoring a polynomial.

### 11, Complex Zeros and the Fundamental theorem of Algebra(4.4)

(1)Complete Factorization Theorem (page 347)

See Example 2 (page 348) for complete factorization.

(2)Zeros Theorem(page 348)

See Example 4 and Example 5 (page 351).

(3)Conjugate Zeros Theorem (page 352)

See Example 6 (page 352) for the theorem.

(4)Linear and Quadratic Factors Theorem

See Example 8 (page 354).

### 12, Rational functions(4.5)

(1)Rational function:  $f(x) = \frac{P(x)}{Q(x)}$

(2)The method to find the horizontal and vertical asymptotes

( $f(x)$  is in the lowest term.): every vertical asymptote corresponds to a factor of  $Q(x)$ ; for the horizontal asymptote, if the degree of  $P(x)$  is less than the

degree of  $Q(x)$ ,  $y = 0$  is the horizontal asymptote; if the degree of  $P(x)$  equals the degree of  $Q(x)$ ,  $y = \frac{a_n}{b_n}$  is the horizontal asymptote, where  $a_n$  and  $b_n$  are the leading coefficients of  $P(x)$  and  $Q(x)$ , respectively.

(3) Graphing rational functions (page 362)

Example: graph  $y = \frac{x^2 - x - 6}{x^2 - x - 2}$

(4) Slant asymptotes and end behavior:

See Example 8 (page 366)

***The second midterm will be at 121 Sparks on Tuesday  
Nov.14 from 6:30PM-7:45PM.***

***GOOD LUCK!!***