

## Review for math22

The first midterm will be at 100 Thomas October 10 from 6:30-7:45PM.

Sample exam: <http://www.math.psu.edu/UG/SampleExamsMath22.htm>

### 1, Basic Equations(1.1)

(1) Linear Equations:  $ax + b = 0$

Example:  $7x - 4 = 3x + 8$ ;  $\frac{x}{6} + \frac{2}{3} = \frac{3}{4}x$

(2) Equations Involving Fractional Expressions

Example:  $\frac{1}{x+1} + \frac{1}{x-2} = \frac{x+3}{x^2-x-2}$

(3) Solving Equations Using Radicals

The equation  $x^n = a$  has the solution

1,  $x = \sqrt[n]{a}$  if  $n$  is odd.

2,  $x = \pm \sqrt[n]{a}$ , if  $n$  is even and  $a \geq 0$

Example:  $(x - 4)^4 = 5$ ,  $5x^{\frac{2}{3}} - 2 = 43$

### 2, Modeling with Equations (1.2)

Guidelines for Modeling with Equations (page 83)

Review the examples in the book.

### 3, Quadratic Equations(1.3)

(1) Two steps to complete the square

Example:  $x^2 - 8x + 9$

(2) The quadratic formula: the solutions to  $ax^2 + bx + c = 0$  are given by  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Example:  $4x^2 + 12x + 3 = 0$

(3) Discriminant:  $b^2 - 4ac$

The discriminant determines the number of real solutions for a quadratic equation. Example: Find a value for  $k$  such that  $x^2 + \sqrt{2}x + k = 0$  has exactly one real solution.

How many real solutions does the equation have?  $x^2 + 4x - 1 = 0$

### 4, Complex Numbers(1.4)

(1) Definition:  $a + bi$

(2) Arithmetic Operations on Complex Numbers

Example:  $(2 + 5i)(4 - i)$

(3) Dividing Complex Numbers: Simplify  $\frac{a+bi}{c+di}$

Example: Express  $\frac{3+5i}{1-2i}$  in the form  $a + bi$

(4) Square Roots of Negative Numbers: the principal square root of  $-r$  for  $r > 0$  is  $\sqrt{-r} = i\sqrt{r}$ .

(5) Complex Roots of Quadratic Equations

Example: Solve the equation:  $x^2 + 4x + 5 = 0$ .

### 5, Other Types of Equations(1.5)

You need to know how to solve the following equations:

- 1)  $x^4 - 8x^2 + 8 = 0$ ; 2)  $\frac{3}{x} + \frac{5}{x+2} = 2$ ; 3)  $3x^4 = -48$   
 4)  $x^3 - x^2 - 2x + 2 = 0$ ; 5)  $2x^{\frac{4}{3}} - x^{\frac{2}{3}} - 6 = 0$ ; 6)  $2x - 1 = -\sqrt{2-x}$

### 6, Inequalities(1.6)

- (1) Properties of Inequalities (page 125)  
 Example: (linear inequalities)  $\frac{x}{3} - \frac{x+1}{2} > 2$ ;  
 (2) Steps for solving polynomial inequalities and inequalities involving a quotient of polynomials. (page 128)  
 Example: 1)  $-x^2 + 2x + 8 > 0$  2)  $\frac{1+x}{1-x} \leq 1$

### 7, Absolute Value Equations and Inequalities(1.7)

- (1)  $|x| = c$  is equivalent to  $x = \pm c$   
 Example:  $3|x - 7| + 5 = 16$   
 (2) Absolute Value Inequalities (page 136)  
 Example:  $|1 - \frac{t}{2}| > 5$

### 8, The Coordinate Plane(2.1)

- (1) Understand the coordinates of a point.  
 Example: sketch the region given by the set  $\{(x, y) ||x| \geq 1\}$   
 (2) The Distance Formula:  $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$   
 Example: 1) Find the distance between (2, 5) and (3, 7).  
 (3) The Midpoint Formula: the midpoint of the line segment from  $(x_1, y_1)$  to  $(x_2, y_2)$  is  $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$ .

### 9, Graphs of Equations in Two Variables(2.2)

- (1) Graphing Equations by Plotting Points (page 160)  
 (2) Intercepts (page 161)  
 Example: Find the  $x$ - and  $y$ -intercepts of the graph of the equation  $y = \sqrt{4-x^2}$   
 (3) Circles:  $(x-h)^2 + (y-k)^2 = r^2$ , where  $(h, k)$  is the center, and  $r$  represents the radius.  
 Example: Find the equation of the circle with radius 3 and center (2, 5).  
 Find the center and radius of the circle:  $x^2 + y^2 + 2x - 6y + 7 = 0$ .  
 (4) Three Types of Symmetry: review pages 164-165 for the symmetry and three tests.  
 Example: test the symmetry for the equations: 1)  $y = x^4 + \frac{1}{x^2} + 1$ ; 2)  $y = x^3 - 3x$ ;  
 3)  $y = |x + 3|$ ; 4)  $(x - 1)^2 + y^2 = 3$

### 10, Equations of lines(2.4)

- (1) Definition: Slope:  $\frac{y_2 - y_1}{x_2 - x_1}$   
 Example: The slope of the line through (-2, 4), (a, 4-a) is 1. What is a?  
 Remarks: 1, The slope of a horizontal line is zero. 2, The slope of a vertical line is undefined.  
 (2) Point-Slope Form:  $y - y_1 = m(x - x_1)$   
 (3) The Slope-Intercept Formula:  $y = mx + b$

Example: Determine the equation of line passing through  $(1, 2), (3, 3)$

(4) Let  $m_1$  and  $m_2$  be the slopes of two lines;

1, the lines are parallel if and only if  $m_1 = m_2$ ;

2, the lines are perpendicular if and only if  $m_1 m_2 = -1$ ;

Example: Find the equation of the line passing  $(4, 5)$  and perpendicular to the line  $2x + y = 9$ .

### 11, Modeling: Variation(2.5)

(1) Direct Variation:  $y = kx$

(2) Inverse variation:  $y = \frac{k}{x}$

(3) Joint Variation:  $z = kxy$  (4)  $z$  is proportional to  $x$  and inversely proportional to  $y$ :  $z = \frac{kx}{y}$