MATH 535

HOMEWORK ASSIGNMENT 9

due on Wednesday, 11/17/04

26. Let E be a finite extension of a field F of prime characteristic p.

(1) Let $\alpha \in E$, and $m(x)$ be the minimal polynomial of α over the field $F(\alpha^p)$. Show that $m(x)$ splits over E, and in fact α is the only root, so that $m(x)$ is a power of $(x - \alpha)$.

(2) If α is separable over the field $F(\alpha^p)$, show that $\alpha \in F(\alpha^p)$.

In the next two problems you will prove transitivity of separable extensions.

27. Let E be a finite extension of a field F of prime characteristic p.

(1) Let $F(E^p)$ be a subfield of E obtained from F by adjoining the p^{th} powers of all elements of E. Show that $F(E^p)$ consists of all finite linear combinations of elements in E^p with coefficients in F.

(2) Assume that $F(E^p) = E$. If elements $y_1, \ldots, y_r$ are linearly independent over F, show that $y_1^p, \ldots, y_r^p$ are linearly independent over F.

28.

(1) Let E be a finite extension of a field F of prime characteristic p. Show that E is separable if and only if $E = F(E^p)$.

(2) If $F \subseteq K \subseteq E$ with $[E : F] < \infty$, with E separable over K and K separable over F, show that E is separable over F.