MATH 535

HOMEWORK ASSIGNMENT 7

due on Monday, 11/01/04

20. Third Isomorphism Theorem for Rings: If R is a commutative ring having ideals $I \subseteq J$, then J/I is an ideal in R/I and there is a ring isomorphism $(R/I)/(J/I) \cong R/J$.

21. Chinese Remainder Theorem:

(i) Prove that if k is a filed and $f(x), g(x) \in k[x]$ are relatively prime, then given $a(x), b(x) \in k[x]$ there exists $c(x) \in k[x]$ with

$$c - a \in (f) \text{ and } c - b \in (g);$$

moreover, if $d(x)$ is another common solution, then $c - d \in (fg)$. **Hint.** Adapt the proof of a Chinese Remained Theorem for integers: Theorem 1.28.

(ii) Prove that if k is a field and $f(x), g(x) \in k[x]$ are relatively prime, then there is a ring isomorphism

$$k[x]/(f(x)g(x)) \cong k[x]/(f(x)) \times k[x]/(g(x)).$$

Hint. See the proof of Theorem 2.81.

22. Let K/k be a field extension, $\alpha \in K$, and $k(\alpha)$ be a subfield of K obtained by adjoining α to k. Prove that

$$k(\alpha) \cong \{f(\alpha)/g(\alpha) \mid f(x), g(x) \in k[x],\ g(\alpha) \neq 0\}.$$