MATH 502: REAL AND COMPLEX ANALYSIS

SPRING 2004

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PROBLEM SET # 7 : March 18

LEBESGUE INTEGRATION

Due on Wednesday 3-24-04

In all problems the measure space is assumed to be finite.

24.We will call a function of the form $\sum_{i=n}^{\infty} t_n \chi_{A_n}$ where $A_1, A_2, \dots$ are disjoint measurable sets *a generalized simple function.* Such a function is called *integrable* if $\sum_{n=1}^{\infty} |t_n| \mu(A_n) < \infty$.

Prove that the uniform limit of a sequence of integrable generalized simple functions is a Lebesgue integrable function.

25. Show that almost everywhere limit of a sequence of integrable generalized simple functions may not be Lebesgue integrable.

26. Show that if a measurable function f is such that for any function $g \in L^1$ the product $f \cdot g$ is Lebesgue integrable then f is essentially bounded (bonded outside a set of measure zero).

27. Prove that for any finite Borel measure μ on the interval $[0, 1]$ polynomials are dense in the space $L^1([0, 1], \mu)$.