MATH 502: REAL AND COMPLEX ANALYSIS

SPRING 2004

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PROBLEM SET # 2: January 22

RIESZ INTEGRAL

Due on Wednesday 1-28-04

5. Consider a Riesz integral l on a compact metric space. Prove that for disjoint closed sets A_1 and A_2 the upper Riemann measure is additive, i.e. the upper integral of $\chi_{A_1 \cup A_2}$ is equal to $\chi_{A_1} + \chi A_2$.

6. Consider a Riesz integral l on a compact metric space. Prove that if the one–point set $\{x\}$ is measurable and not a null-set then x is an *isolated point* in X.

7. Let I^2 be the unit square and l be a Riesz integral on I^2. Prove that for any $r > 0$ there exist a measurable partition of I^2 into rectangles of diameter $\leq r$.
Hint: Use an argument similar to the one which proves measurability of balls.

8. Prove that no interval can be represented as the union of *countably many* null–sets with respect to the standard Riemann measure.

An extra credit problem

2E. Consider a Riesz integral l on a compact metric space. Prove that there are at most countably many points $x \in X$ such that the one–point set $\{x\}$ is not a null-set.
Hint: Use problem N5.