

MATH 502: REAL AND COMPLEX ANALYSIS

SPRING 2004

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PROBLEM SET # 10: April 8

RADON-NIKODYM AND RIESZ REPRESENTATION THEOREMS

Due on Wednesday April 14

36. Suppose μ is a Borel measure on $[0, 1]$ such that for some constant C and every interval $[a, b] \subset [0, 1]$, $\mu([a, b]) \leq C(b - a)$. Prove that $\mu = \rho\lambda$ where ρ is a bounded Borel function and λ Lebesgue measure.

37. Suppose l is a continuous linear functional on the space $C([0, 1])$ such that for some function $f_0 \neq 0$, $l(f_0) = \|l\| \cdot \|f_0\|$. Prove that there exist two disjoint closed sets $A, B \subset [0, 1]$ (one of them may be empty) and measures μ and ν supported on A and B correspondingly such that

$$l(f) = \int_A f d\mu - \int_B f d\nu.$$

38. Let X be a compact metric space without isolated points. Prove that there exists continuum different non-atomic Borel probability measures on X . (A measure μ on X is a *probability* measure if $\mu(X) = 1$).